

1. If $6 : x :: 3 : 15$, find x .
2. Find the fourth proportional to 3,5,27.
3. (i) Find the third proportional to 5,10. (ii) Find the mean proportional between 3 and 75.
4. If $x, 6, 18, y$ are in continued proportion, find x and y .
5. Find the numbers such that the mean proportional between them is 24, and the third proportional to them is 192.
6. What number must be added to each of the four numbers 10,18, 22 and 38 to make them in proportion?
7. If $x : y :: y : z$, prove that $x : z = x^2 : y^2$.
8. If y is the mean proportional between x and z , prove that $xy + yz$ is the mean proportional between $x^2 + y^2$ and $y^2 + z^2$.
9. If q is the mean proportional between p and r , prove that $\left(\frac{pq+qr+rp}{p+q+r}\right)^3 = pqr$.
10. If $a : b = c : d$, show that $(3a - 7b) : (3a + 7b) :: (3c + 7d) : (3c - 7d)$.
11. If $a : b = c : d$, show that $\frac{2a+3b}{2c+3d} = \frac{2a-3b}{2c-3d}$.
12. If $(4a + 5b)(4c = 5d) = (4a-5b)(4c + 5d)$, prove that $\frac{a}{b} = \frac{c}{d}$.
13. If $a = \frac{4xy}{x+y}$, find the value of $\frac{a+2x}{a-2x} + \frac{a+2y}{a-2y}$ by using the properties of proportion.
14. If $\frac{2a+2b-3c-3d}{2a-2b-3c+3d} = \frac{a+b}{a-d-4c+4d}$, then prove that $a : b = c : d$.
15. Solve the following equations for x , using properties of proportion.
 - a. $\frac{\sqrt{5}+\sqrt{5-x}}{\sqrt{5}-\sqrt{5-x}} = 3$
 - b. $\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} = \frac{a}{b}$
16. Solve $\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}} = 4$ for x , using the properties of proportion.
17. If $\frac{\sqrt{2a+1}+\sqrt{2a-1}}{\sqrt{2a+1}-\sqrt{2a-1}}$, show that $x^2 - 4ax + 1 = 0$.
18. Solve for x , using the properties of proportions: $\frac{1+x+x^2}{1-x+x^2} = \frac{62(1+x)}{63(1-x)}$
19. If $x = \frac{\sqrt[3]{m+1}+\sqrt[3]{m-1}}{\sqrt[3]{m+1}-\sqrt[3]{m-1}}$, show that $x^3 - 3mx^2 + 3x - m = 0$.
20. If $\frac{a}{b} = \frac{c}{d}$, show that
 - a. $\frac{a+b}{c+d} = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$
 - b. $(a^4 + c^4) : (b^4 + d^4) :: a^2 c^2 : b^2 d^2$.
21. If $a : b = c : d$, prove that
 - a. $\frac{a^2+c^2}{ab^2+cd^2} = \frac{ab+cd}{b^2+d^2}$.
 - b. $\frac{\sqrt{a^4+c^4}}{\sqrt{b^4+d^4}} = \frac{pa^2+qc^2}{pb^2+qd^2}$.

22. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, show that $\frac{a^3+2c^2e-3ae^2f}{b^4+2d^2f-3bf^3} = \frac{ace}{bdf}$.

23. If $\frac{x+y}{ax+by} = \frac{y+z}{ay+bz} = \frac{z+x}{az+bx}$ $a+y+z \neq 0$, show that each ratio is equal to $\frac{2}{a+b}$.

24. If $\frac{b+c-a}{y+z-x} = \frac{c+a-b}{z+x-y} = \frac{a+b+c}{x+y-z}$, then each ratio is equal to $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$.

25. If $\frac{by+cz}{b^2+c^2} = \frac{cz+ax}{c^2+a^2} = \frac{ax+by}{a^2+b^2}$, then each ratio is equal to $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$.

26. If a, b, c, are in continued proportion, prove that

a. $(a + b) : (b + c) :: a^2 (b - c) : b^2 (a - b)$.

27. If a, b, d, d are in continued proportion, prove that

a. $a : d = (a - b)^3 : (b - c)^3$

b. $(a^2 + b^2 + c^2) (b^2 + c^2 + d^2) (ab + bc + cd)^2$

c. $\sqrt{(a + b + c)(b + c + d)} = \sqrt{ab} + \sqrt{bc} + \sqrt{cd}$

1. If $(3a + 2b) : (5a + 3b) = 18 : 29$, find $a : b$.
If $x : y = 2 : 3$, find the value of $3x + 2y : 2x + 5y$.
2. If $a : b = 5 : 3$, find $(5a + 8b) : (6a - 7b)$.
3. Two numbers are in the ratio $3 : 5$. If 8 is added to each number, the ratio becomes $2 : 3$. Find the numbers.
4. (i) What quantity must be added to each term of the ratio $8 : 15$ so that it becomes equal to $3 : 5$?
(ii) What quantity must be subtracted from each term of the ratio $a : b$ so that it becomes $c : d$?
5. The work done by $(x - 3)$ men in $(2x + 1)$ days and the work done by $(2x + 1)$ men in $(x + 4)$ days are in the ratio $3 : 10$. Find the value of x .
6. When the fare of a certain journey by an airliner was increased in the ratio $5 : 7$ the cost of the ticket for the journey became 1,421. Find the increase in the fare.
7. In a regiment, the ratio of number of officers to the number of soldiers was $3 : 31$ before a battle. In the battle 6 officers and 22 soldiers were killed. The ratio between the number of officers and the number of soldiers now is $1 : 13$. Find the number of officers and soldiers in the regiment before the battle.
8. If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b}$ and $a + b + c = 0$; show that each given ratio is equal to -1 .
9. If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b}$ and $a + b + c \neq 0$; show that each given ratio is equal to $\frac{1}{2}$.
10. Find the compound ratio of :
 - i. $3a : 2b, 2m : n$ and $4x : 3y$
 - ii. $a - b : a + b, a + b^2 : a^2 + b^2$ and $a^4 - b^4 : (a^2 - b^2)^2$.
11. Find the ratio compounded of the duplicate ratio of $5 : 6$, the reciprocal ratio of $25 : 42$ and the sub-triplicate ratio $216 : 343$.
12. Quantities $a, 2, 10$ and b are in continued proportion; find the values of a and b .
13. Find (i) the fourth proportional to $3, 6$ and $4 \cdot 5$.
(ii) the mean proportional between $6 \cdot 25$ and $0 \cdot 16$.
(iii) the third proportional to $1 \cdot 2$ and $1 \cdot 8$.
14. What number should be subtracted from each of the numbers $23, 30, 57$ and 78 ; so that the remainders are in proportion?

15. If $(a^2 + c^2)$, $(ab + cd)$ and $(b^2 + d^2)$ are in continued proportion; prove that a, b, c and d are in proportion.

16. If $p : q :: q : r$, prove that $p : r = p^2 : q^2$.

17. If $a \neq b$ and $a : b$ is the duplicate ratio of $a + c$ and $b + c$, prove that 'c' is the mean proportional between 'a' and 'b'.

18. If $a + c = mb$ and $\frac{1}{b} + \frac{1}{d} = \frac{m}{c}$, prove that a, b, c and d are in proportion.

19. If q is the mean proportional between p and r, prove that : $p^2 - q^2 + r^2 = q^4 \left[\frac{1}{p^2} - \frac{1}{q^2} + \frac{1}{r^2} \right]$.

20. If a, b, c and d are in proportion, prove that :

i.
$$\frac{a-b}{c-d} = \sqrt{\frac{3a^2+8b^2}{3c^2+8d^2}}$$

ii.
$$\frac{5a^2+12c^2}{5b^2+12d^2} = \sqrt{\frac{3a^4-7c^4}{3b^4-7d^4}}$$

21. 6 is the mean proportion between two numbers x and y and 48 is third proportion to x and y. Find the numbers.

22. If $\frac{8x+13y}{8x-13y} = \frac{9}{7}$, find x : y.

23. If $a : b = c : d$, show that : $3a + 2b : 3a - 2b = 3c + 2d : 3c - 2d$.

24. If $\frac{8a-5b}{8c-5d} = \frac{8a+5b}{8c+5d}$, prove that $\frac{a}{b} = \frac{c}{d}$.

25. If $p = \frac{4xy}{x+y}$, find the value of $\frac{p+2x}{p-2x} + \frac{p+2y}{p-2y}$.

26. If $a : b = c : d$; prove that :

$$(a^2 + ac + c^2) : (a^2 - ac + c^2) = (b^2 + bd + d^2) : (b^2 - bd + d^2)$$

27. If x, y and z are in continued proportion, prove that :

$$x^2 - y^2 : x^2 + y^2 = x - z : x + z.$$

28. Using the properties of proportion, solve the following equation for

29. $x : \frac{x^3+3x}{3x^2+1} = \frac{341}{91}$

30. If $x = \frac{\sqrt{3a+2b}\sqrt{3a-2b}}{\sqrt{3a+2b}\sqrt{3a-2b}}$, prove that : $bx^2 - 3ax + b = 0$.