MATHEMATICS 2020

QUESTIONS

(Two Hours and a half)
Answer to this Paper must be written on the paper provided separately.
*You will **not** be allowed to write during the first **15** minutes.*
This time is to be spend in reading the question paper.
The time given at the head of this Paper is the time allowed for writing the answers.
*Attempt **all** questions from **Section A** and **any four** questions from **Section B.***
All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.
Omission of essential working will result in loss of marks.
The intended marks for questions of parts of questions are given in brackets [].
Mathematical table are provided.

SECTION—A (40 Marks)

*(Attempt **all** questions from this Section)*

Question 1.

(a) Solve the following quadratic equation : [3]

$$x^2 - 7x + 3 = 0$$

Give your answer correct to two decimal places.

(b) Given $A = \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix}$ [3]

If $A^2 = 3I$, where I is the identity matrix of order 2, find x and y.

(c) Using ruler and compass, construct a triangle ABC where AB = 3 cm, BC = 4 cm and ∠ABC = 90°. Hence, construct a circle circumscribing the triangle ABC. Measure and write down the radius of the circle. [4]

Question 2.

(a) Use factor theorem to factorise $6x^3 + 17x^2 + 4x - 12$ completely. [3]

(b) Solve the following inequation and represent the solution set on the number line.

$$\frac{3x}{5} + 2 < x + 4 \le \frac{x}{2} + 5, x \in R$$ [3]

(c) Draw a histogram for the given data, using a graph paper : [4]

Weekly Wages (in ₹)	No. of People
3000–4000	4
4000–5000	9
5000–6000	18
6000–7000	6
7000–8000	7
8000–9000	2
9000–10000	4

Estimate the mode from the graph.

Question 3.

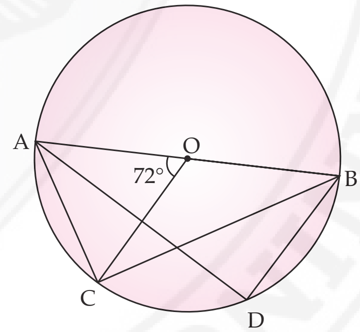
(a) In the figure given below, O is the centre of the circle and AB is a diameter. [3]

If AC = BD and ∠AOC = 72°. Find :

(i) ∠ABC

(ii) ∠BAD

(iii) ∠ABD



(b) Prove that : [3]

$$\frac{\sin A}{1+\cot A} - \frac{\cos A}{1+\tan A} = \sin A - \cos A$$

(c) In what ratio is the line joining P (5, 3) and Q (– 5, 3) divided by the y-axis ? Also find the coordinates of the point of intersection. [4]

Question 4.

(a) A solid spherical ball of radius 6 cm is melted and recast into 64 identical spherical marbles. Find the radius of each marble. [3]

(b) Each of the letters of the word 'AUTHORIZES' is written on identical circular discs and put in a bag.

They are well shuffled. If a disc is drawn at random from the bag, what is the probability that the letter is : **[3]**

(i) a vowel

(ii) one of the first 9 letters of the English alphabet which appears in the given word.

(iii) one of the last 9 letters of the English alphabet which appears in the given word ?

(c) Mr. Bedi visits the market and buys the following articles : **[4]**

Medicines costing ₹ 950, GST @ 5%

A pair of shoes costing ₹ 3000, GST @ 18%

A laptop bag costing ₹ 1000 with a discount of 30%, GST @ 18%.

(i) Calculate the total amount of GST paid.

(ii) The total bill amount including GST paid by Mr. Bedi.

SECTION—B (40 Marks)

(Attempt any **four** questions from this Section)

Question 5.

(a) A company with 500 shares of nominal value ₹ 120 declares an annual dividend of 15%. Calculate : **[3]**

(i) the total amount of dividend paid by the company.

(ii) annual income of Mr. Sharma who holds 80 shares of the company.

If the return percent of Mr. Sharma from his shares is 10%, find the market value of each share.

(b) The mean of the following data is 16. Calculate the value of f. **[3]**

Marks	5	10	15	20	25
No. of Students	3	7	f	9	6

(c) The 4^{th}, 6^{th} and the last term of a geometric progression are 10, 40 and 640 respectively. If the common ratio is positive, find the first term, common ratio and the number of terms of the series. **[4]**

Question 6.

(a) If $A = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$ **[3]**

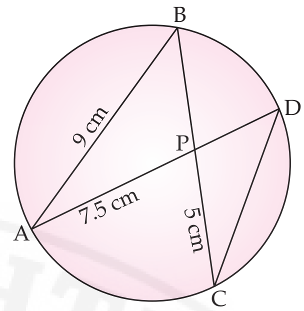
Find $A^2 - 2AB + B^2$

(b) In the given figure AB = 9 cm, PA = 7.5 cm and PC = 5 cm. Chords AD and BC intersect at P. **[3]**

(i) Prove that $\Delta PAB \sim \Delta PCD$

(ii) Find the length of CD.

(iii) Find area of ΔPAB : area of ΔPCD



(c) From the top of a cliff, the angle of depression of the top and bottom of a tower are observed to be 45° and 60° respectively. If the height of the tower is 20 m. **[4]**

Find :

(i) the height of the cliff

(ii) the distance between the cliff and the tower.

Question 7.

(a) Find the value of 'p' if the lines, $5x - 3y + 2 = 0$ and $6x - py + 7 = 0$ are perpendicular to each other. Hence, find the equation of a line passing through $(-2, -1)$ and parallel to $6x - py + 7 = 0$. **[3]**

(b) Using properties of proportion find $x : y$, given :

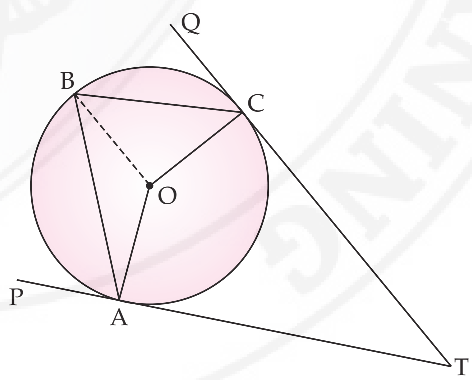
$$\frac{x^2 + 2x}{2x + 4} = \frac{y^2 + 3y}{3y + 9}$$ **[3]**

(c) In the given figure TP and TQ are two tangents to the circle with centre O, touching at A and C respectively. If ∠BCQ = 55° and ∠BAP = 60°, find : **[4]**

(i) ∠OBA and ∠OBC

(ii) ∠AOC

(iii) ∠ATC



Question 8.

(a) What must be added to the polynomial $2x^3 - 3x^2 - 8x$, so that it leaves a remainder 10 when divided by $2x + 1$? **[3]**

(b) Mr. Sonu has a recurring deposit account and deposits ₹ 750 per month for 2 years. If he gets ₹ 19125 at the time of maturity, find the rate of interest. **[3]**

(c) *Use graph paper for this question.* **[4]**

Take 1 cm = 1 unit on both x and y axes.

(i) *Plot the following points on your graph sheets : A (– 4, 0), B (–3, 2), C (0, 4), D (4, 1) and E (7, 3)*

(ii) *Reflect the points B, C, D and E on the x-axis and name them as B′, C′, D′ and E′ respectively.*

(iii) *Join the points A, B, C, D, E, E′, D′, C′, B′ and A in order.*

(iv) *Name the closed figure formed.*

Question 9.

(a) *40 students enter for a game of shot-put competition. The distance thrown (in metres) is recorded below :* **[6]**

Distance in m	Number of Students
12 – 13	3
13 – 14	9
14 – 15	12
15 – 16	9
16 – 17	4
17 – 18	2
18 – 19	1

Use a graph paper to draw an ogive for the above distribution.

Use a scale of 2 cm = 1 m on one axis and 2 cm = 5 students on the other axis.

Hence using your graph find :

(i) *the median*

(ii) *Upper Quartile*

(iii) *Number of students who cover a distance which is above $16\frac{1}{2}$ m.*

(b) *If* $x = \frac{\sqrt{2a+1}+\sqrt{2a-1}}{\sqrt{2a+1}-\sqrt{2a-1}}$, *prove that* $x^2 - 4ax + 1 = 0$ **[4]**

Question 10.

(a) *If the 6th term of an A.P. is equal to four times its first term and the sum of first six terms is 75, find the first term and the common difference.* **[3]**

(b) *The difference of two natural numbers is 7 and their product is 450. Find the numbers.* **[3]**

(c) *Use ruler and compass for this question. Construct a circle of radius 4.5 cm. Draw a chord AB = 6 cm.* **[4]**

(i) *Find the locus of points equidistant from A and B. Mark the point where it meets the circle as D.*

(ii) *Join AD and find the locus of points which are equidistant from AD and AB. Mark the point where it meets the circle as C.*

(iii) *Join BC and CD. Measure and write down the length of side CD of the quadrilateral ABCD.*

Question 11.

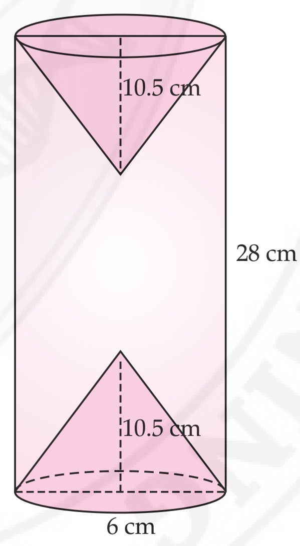
(a) *A model of a high rise building is made to a scale of 1 : 50.* **[3]**

(i) *If the height of the model is 0.8 m, find the height of the actual building.*

(ii) *If the floor area of a flat in the building is 20 m², find the floor area of that in the model.*

(b) *From a solid wooden cylinder of height 28 cm and diameter 6 cm, two conical cavities are hollowed out. The diameters of the cones are also of 6 cm and height 10.5 cm.* **[3]**

Taking $\pi = \frac{22}{7}$ *find the volume of the remaining solid.*



(c) *Prove the identity* **[4]**

$$\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2 = \tan^2\theta$$

ANSWERS

SECTION—A

Solution 1.

(a) Given : $x^2 - 7x + 3 = 0$

Here, $a = 1, b = -7$ and $c = 3$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-7) \pm \sqrt{(-7)^2 - 4\times1\times3}}{2\times1}$$

$$= \frac{7 \pm \sqrt{49 - 12}}{2}$$

$$= \frac{7 \pm \sqrt{37}}{2} = \frac{7 \pm 6.08}{2}$$

Taking positive sign,

$$x = \frac{7 + 6.08}{2} = 6.541$$

Taking negative sign,

$$x = \frac{7 - 6.08}{2} = 0.458$$

Hence, $x = 6.54$ and 0.46 **Ans.**

(b) Given : $A = \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix}$

Also, $A^2 = 3I$

$$\Rightarrow \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix}\begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix} = 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x^2 + 3y & 3x + 9 \\ xy + 3y & 3y + 9 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

Comparing both sides, we get

$$3x + 9 = 0$$

$$\Rightarrow \quad x = -\frac{9}{3} = -3$$

and $3y + 9 = 3$

$$\Rightarrow \quad 3y = 3 - 9 = -6$$

$$\Rightarrow \quad y = -\frac{6}{3} = -2$$

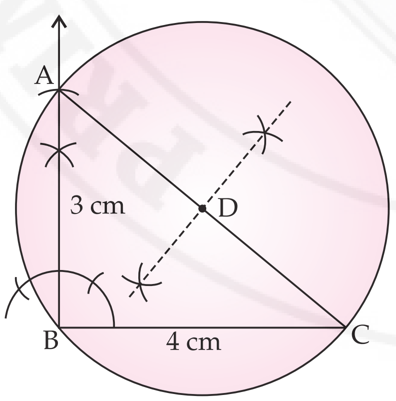
$\therefore$ $x = -3$ and $y = -2$ **Ans.**

(c) Steps of construction:

(i) Draw $BC = 4$ cm.

(ii) Make an angle of 90° at B and cut an arc of radius 3 cm on it to get point A.

(iii) Join AC. Thus, ΔABC is obtained.



(iv) Draw perpendicular bisector of AC, which meets AC at D.

(v) Taking D as centre and radius equal to AD or DC, draw a circle. Thus, it is a required circle.

Since, $AC = 5$ cm, so, $AD = 2.5$ cm.

Solution 2.

(a) Let $p(x) = 6x^3 + 17x^2 + 4x - 12$

$$\because \quad p(-2) = 6 \times (-2)^3 + 17 \times (-2)^2 + 4(-2) - 12$$
$$= 6 \times (-8) + 17 \times 4 - 8 - 12$$
$$= -48 + 68 - 20$$
$$= -68 + 68 = 0$$

$\therefore$ $(x + 2)$ is a factor of $p(x)$

Dividing $p(x)$ by $(x + 2)$, see get

$$x + 2)\overline{\,6x^3 + 17x^2 + 4x - 12\,}(6x^2 + 5x - 6$$
$$6x^3 + 12x^2$$
$$- \quad -$$
$$5x^2 + 4x$$
$$5x^2 + 10x$$
$$- \quad -$$
$$-6x - 12$$
$$-6x - 12$$
$$+ \quad +$$
$$0$$

Now for the quotient

$$\because \quad 6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$$
$$= 3x(2x + 3) - 2(2x + 3)$$
$$= (3x - 2)(2x + 3)$$

Therefore,

$$6x^3 + 17x^2 + 4x - 12 = (x + 2)(6x^2 + 5x - 6)$$
$$= (x + 2)(2x + 3)(3x - 2)$$

Ans.

(b) Given : $\frac{3x}{5} + 2 < x + 4 \le \frac{x}{2} + 5$

Now, $\frac{3x}{5} + 2 < x + 4$

$$\Rightarrow \quad \frac{3x}{5} - x < 4 - 2$$

$$\Rightarrow \quad \frac{3x - 5x}{5} < 2$$

$$\Rightarrow \quad -2x < 10$$

$$\Rightarrow \quad x > -\frac{10}{2}$$

$$\Rightarrow \quad x > -5$$

And, $x + 4 \le \frac{x}{2} + 5$

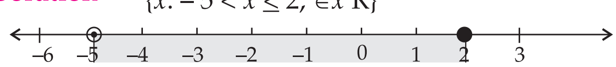
$$\Rightarrow \quad x - \frac{x}{2} \le 5 - 4$$

$$\Rightarrow \quad \frac{x}{2} \le 1$$

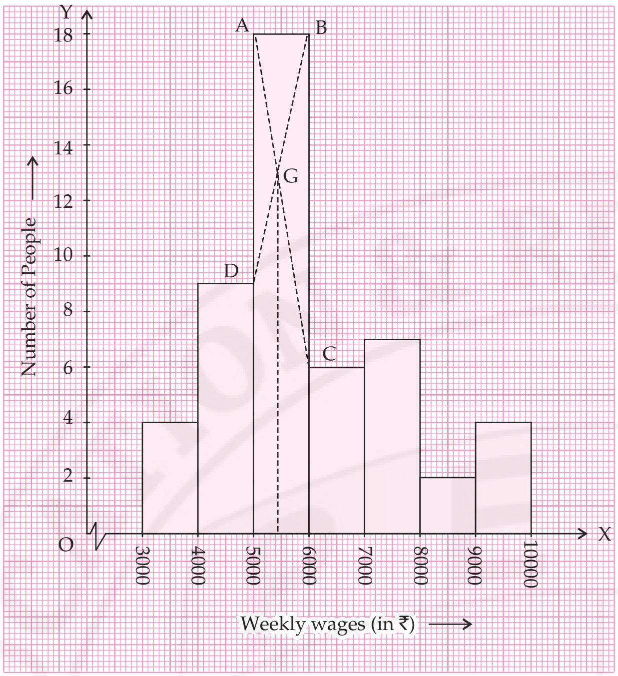
$$\Rightarrow \quad x \le 2$$

Hence, $-5 < x \le 2$

Solution $\{x : -5 < x \le 2, \in x \text{ R}\}$



(c)



We have, maximum frequency = 18

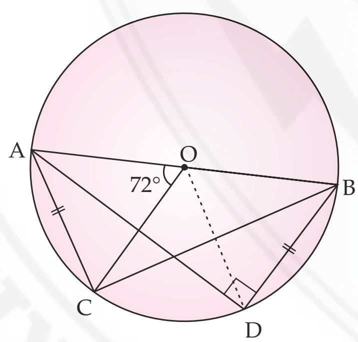
∴ Modal class = 5000 – 6000

Join AC and BD and draw a perpendicular from point G to X-axis at 5450.

Hence, estimated mode is 5450. **Ans.**

Solution 3.

(a) Given : $AC = BD$ and $\angle AOC = 72°$



(i) ∵ Angle subtended by an arc at the centre is twice the angle subtended by the same arc at any point on the remaining part of the circle.

∴ $\angle AOC = 2\angle ABC$

$\Rightarrow \angle ABC = \frac{1}{2} \angle AOC$

$= \frac{1}{2} \times 72° = 36°$ **Ans.**

(ii) Since $BD = AC$ and equal chords subtend equal angles at the centre.

So, $\angle BOD = \angle AOC = 72°$

and, $\angle BAD = \frac{1}{2} \angle BOD$

$= \frac{1}{2} \times 72° = 36°$ **Ans.**

(iii) In ΔABD,

$\angle BAD + \angle ABD + \angle ADB = 180°$

$\Rightarrow 36° + \angle ABD + 90° = 180°$

[∵ $\angle ADB$ is in semicircle and angles of a triangle adds upto 180°]

$\Rightarrow \angle ABD = 180° - 126°$

$= 54°$ **Ans.**

(b) To prove :

$$\frac{\sin A}{1+\cot A} - \frac{\cos A}{1+\tan A} = \sin A - \cos A$$

Taking, L.H.S. $= \frac{\sin A}{1+\cot A} - \frac{\cos A}{1+\tan A}$

$$= \frac{\sin A \times \sin A}{\sin A + \cos A} - \frac{\cos A \times \cos A}{\cos A + \sin A}$$

$$= \frac{\sin^2 A - \cos^2 A}{\sin A + \cos A}$$

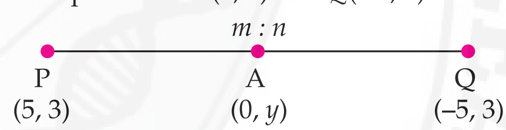
$$= \frac{(\sin A - \cos A)(\sin A + \cos A)}{(\sin A + \cos A)}$$

[∵ $a^2 - b^2 = (a - b)(a + b)$]

$= \sin A - \cos A =$ R.H.S.

Hence Proved.

(c) Given points are $P(5, 3)$ and $Q(-5, 3)$



Let the coordinates of the point where this line meets y-axis be $A(0, y)$ and the ratio be $m : n$. Using section formula, we have

$$(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$\Rightarrow 0 = \frac{-5m + 5n}{m+n}$

$\Rightarrow 0 = -5m + 5n$

$\Rightarrow 5m = 5n$

or $m : n = 5 : 5 = 1 : 1$

Now, $y = \frac{m \times 3 + 3 \times n}{m+n}$

$= \frac{1 \times 3 + 1 \times 3}{1+1}$

$= \frac{3+3}{2}$

$= \frac{6}{2} = 3$

Hence, the required ratio is 1 : 1 and the point of intersection is (0, 3). **Ans.**

Solution 4.

(a) Let the radius of each spherical marble be r cm.

Then, volume of 64 spherical marbles

= volume of solid spherical ball

$$\Rightarrow \quad 64 \times \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 6^3$$

$$\Rightarrow \quad r^3 = \frac{6 \times 6 \times 6}{64}$$

$$\Rightarrow \quad r^3 = \left(\frac{6}{4}\right)^3$$

$$\text{or} \quad r = \frac{6}{4} = 1.5$$

Hence, the radius of each marble is 1.5 cm. **Ans.**

(b) Letters are $A, U, T, H, O, R, I, Z, E, S$.

$\Rightarrow$ Total number of letters in the given word = 10.

(i) Here, vowels are A, U, O, I, E.

$\Rightarrow$ Number of vowels = 5

So, probability (a vowel) = $\frac{5}{10} = \frac{1}{2}$ **Ans.**

(ii) Letters in the given word which are in first 9 letters of english alphabets are A, I, E and H.

$\Rightarrow$ Number of such letters = 4

$\therefore$ Probability = $\frac{4}{10} = \frac{2}{5}$ **Ans.**

(iii) Letters in the given word which are in last 9 letters of english alphabets are U, T, R, Z and S.

$\Rightarrow$ Number of such letters = 5

$\therefore$ Probability = $\frac{5}{10} = \frac{1}{2}$ **Ans.**

(c) **(i)** Cost of medicines = ₹ 950

GST on medicines = 5% of 950

$= \frac{5}{100} \times 950$

= ₹ 47.50

Cost of a pair of shoes = ₹ 3000

GST on shoes = 18% of ₹ 3000

$= \frac{18}{100} \times 3000$

= ₹ 540

Cost of laptop bag = ₹ 1000

Discount on bag = 30% of 1000

$= \frac{30}{100} \times 1000$

= ₹ 300

$\therefore$ Cost of laptop bag after discount

= ₹ (1000 – 300)

= ₹ 700

GST on laptop bag = 18% of ₹ 700

$= \frac{18}{100} \times 700$

= ₹ 126

$\therefore$ Total GST on all items

= ₹ (47.50 + 540 + 126)

= ₹ 713.50 **Ans.**

(ii) Total bill including GST = cost of (medicines + shoes + laptop bag) + Total GST on all items.

= ₹ (950 + 3000 + 700) + ₹ 713.50

= ₹ (4650 + 713.50)

= ₹ 5363.50 **Ans.**

SECTION—B

Solution 5.

(a) **(i)** Total number of shares = 500

Nominal value of each share = ₹ 120

And, Dividend = 15%

Total value of shares = ₹ (500 × 120)

= ₹ 60,000

So, Total dividend = 15% of ₹ 60,000

$= \frac{15}{100} \times 60{,}000$

= ₹ 9,000 **Ans.**

(ii) Annual income of 80 shares

= 15% of (80 × 120)

$= \frac{15}{100} \times 9600$

= ₹ 1,440 **Ans.**

Let the market value of each share be ₹ x.

So, 10% of $80x$ = 1440

$$\Rightarrow \quad \frac{10}{100} \times 80x = 1440$$

$$\Rightarrow \quad x = \frac{1440 \times 10}{80}$$

$$x = 180$$

So, the market value of each share is ₹ 180. **Ans.**

(b)

Marks x_i	No. of students f_i	$f_i x_i$
5	3	15
10	7	70
15	f	$15f$
20	9	180
25	6	150
	$\sum f_i = 25 + f$	$\sum f_i x_i = 415 + 15f$

We know, mean $= \frac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow \quad 16 = \frac{415 + 15f}{25 + f}$

$\Rightarrow \quad 400 + 16f = 415 + 15f$

$\Rightarrow \quad 16f - 15f = 415 - 400$

$\Rightarrow \quad f = 15$ **Ans.**

(c) Given : $a_4 = 10, a_6 = 40, a_n = 640$

$\therefore \quad ar^3 = 10$ ...(i)

and $\quad ar^5 = 40$ ...(ii)

On dividing (ii) by (i),

$$\frac{ar^5}{ar^3} = \frac{40}{10}$$

$\Rightarrow \quad r^2 = 4$

$\Rightarrow \quad r = 2$ [$\because$ r is positive]

Putting $r = 2$ in equation (i), we get

$a \times 2^3 = 10$

$\Rightarrow \quad a = \frac{10}{8} = \frac{5}{4}$

Now, $\quad a_n = 640$

$\Rightarrow \quad ar^{n-1} = 640$

$\Rightarrow \quad \frac{5}{4} \times (2)^{n-1} = 640$

$\Rightarrow \quad 2^{n-1} = \frac{640 \times 4}{5} = 128 \times 4$

$\Rightarrow \quad 2^{n-1} = 2^9$

$\therefore \quad n - 1 = 9$

$\therefore \quad n = 9 + 1 = 10$

Hence, $a = \frac{5}{4}$, $r = 2$ and $n = 10$ **Ans.**

Solution 6.

(a) Given : $A = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$

Now, $A^2 - 2AB + B^2 = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}$

$$-2\begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9+0 & 0+0 \\ 15+5 & 0+1 \end{bmatrix} - 2\begin{bmatrix} -12+0 & 6+0 \\ -20+1 & 10+0 \end{bmatrix}$$

$$+ \begin{bmatrix} 16+2 & -8+0 \\ -4+0 & 2+0 \end{bmatrix}$$

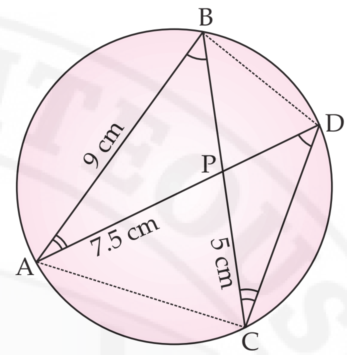
$$= \begin{bmatrix} 9 & 0 \\ 20 & 1 \end{bmatrix} - 2\begin{bmatrix} -12 & 6 \\ -19 & 10 \end{bmatrix} + \begin{bmatrix} 18 & -8 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 \\ 20 & 1 \end{bmatrix} + \begin{bmatrix} 24 & -12 \\ 38 & -20 \end{bmatrix} + \begin{bmatrix} 18 & -8 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9+24+18 & 0-12-8 \\ 20+38-4 & 1-20+2 \end{bmatrix}$$

$$= \begin{bmatrix} 51 & -20 \\ 54 & -17 \end{bmatrix}$$ **Ans.**

(b) Given : $AB = 9$ cm, $PA = 7.5$ cm and $PC = 5$ cm



(i) In ΔPAB and ΔPCD,

$\angle ABC = \angle ADC$

[Angles made by same arc AC or angles in the some segment are equal]

$\angle BAD = \angle BCD$

[Angles made by same arc BD]

$\therefore \quad \Delta PAB \sim \Delta PCD$

[By AA similarity axiom]

Hence Proved.

(ii) $\because$ Ratio of corresponding sides of similar triangles is equal.

$\therefore \quad \frac{AB}{CD} = \frac{PA}{PC} = \frac{PB}{PD}$

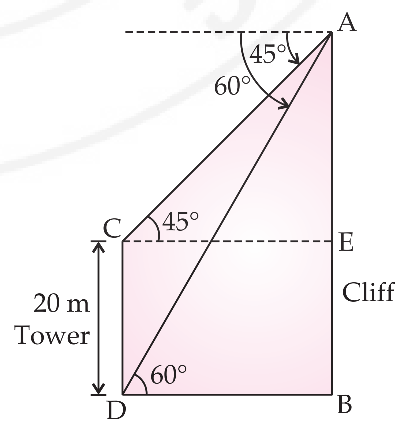
$\Rightarrow \quad \frac{9}{CD} = \frac{7.5}{5} = \frac{PB}{PD}$

$\Rightarrow \quad CD = \frac{9 \times 5}{7.5} = 6$ cm **Ans**

(iii) $\frac{ar\,(\Delta PAB)}{ar\,(\Delta PCD)} = \frac{AB^2}{CD^2}$

$= \frac{9^2}{6^2} = \frac{81}{36} = \frac{9}{4}$ **Ans.**

(c) Let AB be the cliff and CD be the tower.



Also, let $DB = CE = x$ m and $AB = h$ m

(i) In ΔABD,

$$\tan 60° = \frac{AB}{DB}$$

$$\sqrt{3} = \frac{h}{x}$$

$$h = x\sqrt{3} \quad ...(i)$$

And, in ΔACE,

$$\tan 45° = \frac{AE}{CE}$$

$$\Rightarrow \quad 1 = \frac{AB - BE}{x}$$

$$\Rightarrow \quad 1 = \frac{h - 20}{x}$$

$$x = h - 20 \quad ...(ii)$$

Putting the value of x in equation (i), we get

$$h = (h - 20)\sqrt{3}$$

$$\Rightarrow \quad h = \sqrt{3}\,h - 20\sqrt{3}$$

$$\Rightarrow \quad \sqrt{3}\,h - h = 20\sqrt{3}$$

$$\Rightarrow \quad h(\sqrt{3} - 1) = 20\sqrt{3}$$

$$\Rightarrow \quad h = \frac{20\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{20\sqrt{3}(\sqrt{3}+1)}{3-1}$$

$$= 10(3 + \sqrt{3}) = 10(3 + 1.732)$$

$$= 10 \times 4.732$$

$$= 47.32 \text{ m}$$

Hence, the height of cliff is 47.32 m. **Ans.**

(ii) Putting the value of h in equation (ii), we get

$$x = h - 20 = 47.32 - 20$$

$$= 27.32$$

Hence, the distance between the cliff and the tower is 27.32 m. **Ans.**

Solution 7.

(a) Given lines are,

$$5x - 3y + 2 = 0$$

and $6x - py + 7 = 0$

Now, $5x - 3y + 2 = 0$

$$\Rightarrow \quad 3y = 5x + 2$$

$$\Rightarrow \quad y = \frac{5}{3}x + \frac{2}{3}$$

$$\therefore \quad \text{Slope } (m_1) = \frac{5}{3}$$

and $6x - py + 7 = 0$

$$\Rightarrow \quad py = 6x + 7$$

$$\Rightarrow \quad y = \frac{6}{p}x + \frac{7}{p}$$

$$\therefore \quad \text{Slope } (m_2) = \frac{6}{p}$$

Since, given lines are perpendicular to each other,

So, $m_1 \times m_2 = -1$

$$\frac{5}{3} \times \frac{6}{p} = -1$$

$$\Rightarrow \quad p = -10$$

Now, slope $(m_2) = \frac{6}{p} = \frac{6}{-10} = -\frac{3}{5}$

∵ Slopes of parallel lines are equal.

So, slope of required line is $\left(-\frac{3}{5}\right)$.

Now, equation of required line is

$$\frac{y - y_1}{x - x_1} = m$$

$$\Rightarrow \quad \frac{y+1}{x+2} = -\frac{3}{5}$$

$$\Rightarrow \quad 5y + 5 = -3x - 6$$

$$\Rightarrow \quad 3x + 5y + 5 + 6 = 0$$

$$\Rightarrow \quad 3x + 5y + 11 = 0$$

Ans.

(b) Given : $\frac{x^2 + 2x}{2x + 4} = \frac{y^2 + 3y}{3y + 9}$

Using componendo and dividendo,

$$\frac{x^2 + 2x + 2x + 4}{x^2 + 2x - 2x - 4} = \frac{y^2 + 3y + 3y + 9}{y^2 + 3y - 3y - 9}$$

$$\Rightarrow \quad \frac{x^2 + 4x + 4}{x^2 - 4} = \frac{y^2 + 6y + 9}{y^2 - 9}$$

$$\Rightarrow \quad \frac{(x+2)^2}{(x-2)(x+2)} = \frac{(y+3)^2}{(y-3)(y+3)}$$

$$\Rightarrow \quad \frac{x+2}{x-2} = \frac{y+3}{y-3}$$

Again, using componendo and dividendo

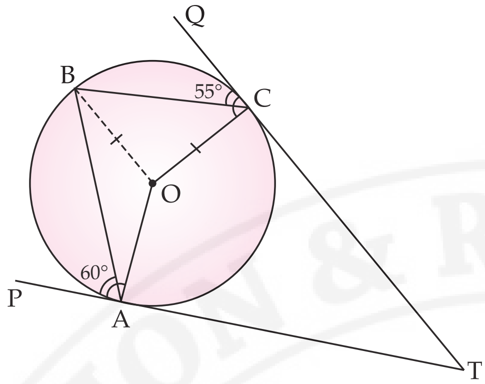
$$\frac{x+2+x-2}{x+2-x+2} = \frac{y+3+y-3}{y+3-y+3}$$

$$\Rightarrow \quad \frac{2x}{4} = \frac{2y}{6}$$

$$\Rightarrow \quad \frac{x}{y} = \frac{4}{6} = \frac{2}{3}$$

Hence, $x : y = 2 : 3$ **Ans.**

(c) Given : $\angle BCQ = 55°$ and $\angle BAP = 60°$



(i) $\angle OAP = 90°$ [$\because$ Tangent is $\perp$ to radius]

$\Rightarrow \angle OAB + \angle PAB = 90°$

$\Rightarrow \angle OAB + 60° = 90°$

$\Rightarrow \angle OAB = 90° - 60° = 30°$

Now, in ΔAOB

$OA = OB$ [Radii of same circle]

$\therefore \angle OBA = \angle OAB = 30°$

[Equal angles opposite to equal sides]

Now, $\angle OCQ = 90°$ [$\because$ Tangent $\perp$ radius]

$\Rightarrow \angle OCB + \angle BCQ = 90°$

$\Rightarrow \angle OCB + 55° = 90°$

$\Rightarrow \angle OCB = 90° - 55° = 35°$

In ΔBOC,

$OC = OB$ [Radii of same circle]

$\Rightarrow \angle OBC = \angle OCB = 35°$ **Ans.**

(ii) We know, angle subtended by an arc at the centre is double the angle subtended on the remaining part of the circle.

$\therefore \angle AOC = 2\angle ABC$

$= 2(\angle OBA + \angle OBC)$

$= 2(30° + 35°)$

$= 2 \times 65° = 130°$ **Ans.**

(iii) In quad. $AOCT$,

$\angle ATC + \angle OAT + \angle AOC + \angle OCT = 360°$

$\Rightarrow \angle ATC + 90° + 130° + 90° = 360°$

[$\because \angle OAT = \angle OCT = 90°$]

$\Rightarrow \angle ATC = 360° - 310° = 50°$ **Ans.**

Solution 8.

(a) Let k be the required term to be added.

So, $p(x) = 2x^3 - 3x^2 - 8x + k$

$\because$ $p(x)$ leaves remainder 10 when divided by $2x + 1$,

$\therefore \quad p\left(-\frac{1}{2}\right) = 10$

$\Rightarrow 2 \times \left(-\frac{1}{2}\right)^3 - 3 \times \left(-\frac{1}{2}\right)^2 - 8 \times \left(-\frac{1}{2}\right) + k = 10$

$\Rightarrow 2 \times \left(-\frac{1}{8}\right) - 3 \times \frac{1}{4} + 4 + k = 10$

$\Rightarrow -\frac{1}{4} - \frac{3}{4} + 4 + k = 10$

$\Rightarrow k = 10 - 4 + \frac{1+3}{4}$

$\Rightarrow k = 6 + 1 = 7$

$\therefore k = 7$ **Ans.**

(b) Here, P = ₹ 750, n = 2 years = 24 months and M.V. = ₹ 19125

We know, $\text{M.V.} = P \times n + \frac{P \times n(n+1)}{2 \times 12} \times \frac{r}{100}$

$\Rightarrow 19125 = 750 \times 24 + \frac{750 \times 24(24+1)}{2 \times 12} \times \frac{r}{100}$

$\Rightarrow 19125 = 18000 + 750 \times 25 \times \frac{r}{100}$

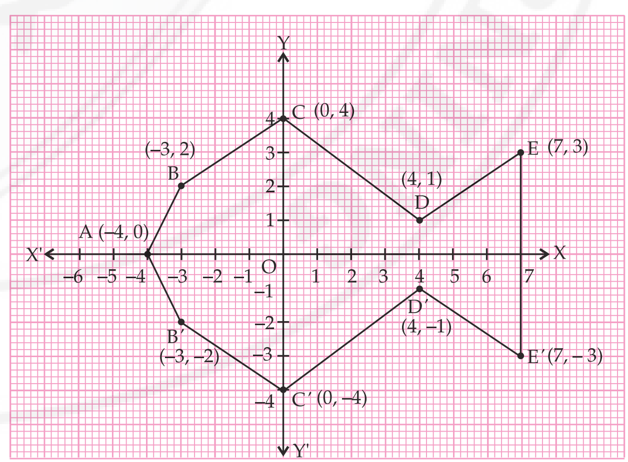
$\Rightarrow 19125 - 18000 = \frac{750 \times r}{4}$

$\Rightarrow 1125 = \frac{750 \times r}{4}$

$\Rightarrow r = \frac{1125 \times 4}{750} = 6$

Hence, the rate of interest is 6% p.a. **Ans.**

(c)



Note : Instead of 1 cm = 1 unit, we have used 0.5 cm = 1 unit on both axes.

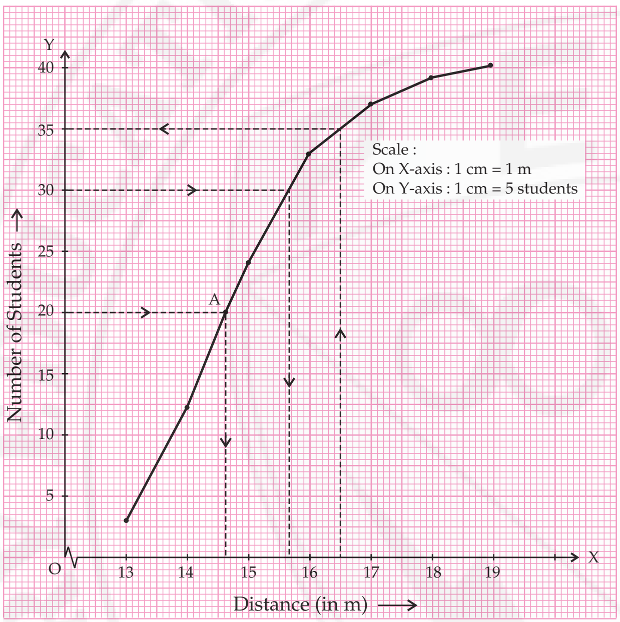
(i), (ii) and **(iii)** see graph.

(iv) Nonagon (irregular), polygon fish

Solution 9.

(a)

Distance in m	Frequency (*f*)	*c.f.*
12 – 13	3	3
13 – 14	9	12
14 – 15	12	24
15 – 16	9	33
16 – 17	4	37
17 – 18	2	39
18 – 19	1	40



Note : Instead of 2 cm = 1 m and 2 cm = 5 students, we have used 1 cm = 1 m and 1 cm = 5 students on X and Y axes, respectively.

(i) Median $= \left(\frac{N}{2}\right)^{th}$ term

$= \left(\frac{40}{2}\right)^{th}$ term

= 20th term

On the graph, through a point 20 on *y*-axis, draw a horizontal line which meets the ogive at point *A*. Through *A*, draw a vertical line which meets the *x*-axis at 14.7.

$\therefore$ Median = 14.7 **Ans.**

(ii) Upper quartile (Q_3) $= \left(\frac{3N}{4}\right)^{th}$ term

$= \left(\frac{3 \times 40}{4}\right)^{th}$ term

= 30th term

= 15.7 **Ans.**

(iii) Number of students who cover more than $16\frac{1}{2}$m = 40 – 35 = 5 **Ans.**

(b) Given : $x = \frac{\sqrt{2a+1}+\sqrt{2a-1}}{\sqrt{2a+1}-\sqrt{2a-1}}$

Using componendo and dividendo,

$$\frac{x+1}{x-1} = \frac{\sqrt{2a+1}+\sqrt{2a-1}+\sqrt{2a+1}-\sqrt{2a-1}}{\sqrt{2a+1}+\sqrt{2a-1}-\sqrt{2a+1}+\sqrt{2a-1}}$$

$$\Rightarrow \frac{x+1}{x-1} = \frac{2\sqrt{2a+1}}{2\sqrt{2a-1}}$$

$$\Rightarrow \left(\frac{x+1}{x-1}\right)^2 = \left(\frac{\sqrt{2a+1}}{\sqrt{2a-1}}\right)^2$$

[Squaring on both sides]

$$\Rightarrow \frac{x^2+1+2x}{x^2+1-2x} = \frac{2a+1}{2a-1}$$

Again, using componendo and dividendo,

$$\frac{x^2+1+2x+x^2+1-2x}{x^2+1+2x-x^2-1+2x} = \frac{2a+1+2a-1}{2a+1-2a+1}$$

$$\Rightarrow \frac{2(x^2+1)}{4x} = \frac{4a}{2}$$

$$\Rightarrow \frac{x^2+1}{2x} = 2a$$

$$\Rightarrow x^2+1 = 4ax$$

$$\Rightarrow x^2 - 4ax + 1 = 0$$

Hence Proved.

Solution 10.

(a) Let the first term of an A.P. be *a* and the common difference be *d*.

$\because$ $a_6 = 4a$ [Given]

$\Rightarrow$ $a + 5d = 4a$

$\Rightarrow$ $5d = 3a$

$\therefore$ $a = \frac{5d}{3}$ ...(i)

Also, $S_6 = 75$ [Given]

$\Rightarrow$ $\frac{6}{2}[2a + (6-1)d] = 75$

$\Rightarrow$ $3\left[2 \times \frac{5d}{3} + 5d\right] = 75$ [Using (i)]

$\Rightarrow$ $3\left[\frac{10d+15d}{3}\right] = 75$

$\Rightarrow$ $25d = 75$

$\therefore$ $d = \frac{75}{25} = 3$

$\therefore$ $a = \frac{5d}{3} = \frac{5 \times 3}{3} = 5$

Hence, $a = 5$ and $d = 3$ **Ans.**

(b) Let the two natural numbers be *x* and *y* such that $x > y$.

Then, $x - y = 7$

$\Rightarrow$ $x = 7 + y$ ...(i)

and $xy = 450$

$\Rightarrow$ $(7 + y)y = 450$ [Using (i)]

$\Rightarrow$ $y^2 + 7y - 450 = 0$

$\Rightarrow$ $y^2 + 25y - 18y - 450 = 0$ (on factorisation)

$\Rightarrow$ $y(y + 25) - 18(y + 25) = 0$

$\Rightarrow$ $(y + 25)(y - 18) = 0$

$\Rightarrow$ $y = -25$ [Neglected]

or $y = 18$

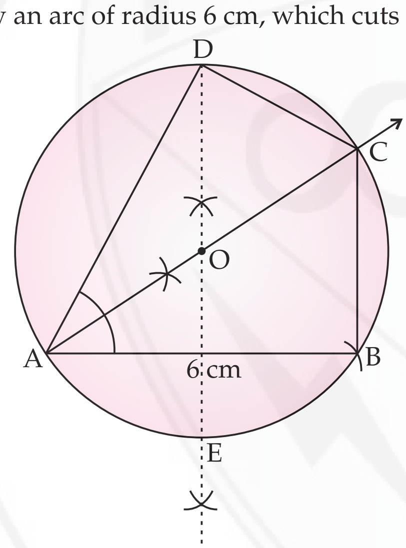
$\therefore$ $y = 18$

$\therefore$ $x = 7 + 18 = 25$

Hence, the numbers are 25 and 18. **Ans.**

(c) Steps of construction :

1. Draw a circle of radius 4.5 cm.
2. Take a point *A* on the circle. Taking *A* as centre, draw an arc of radius 6 cm, which cuts circle at *B*.



3. Join *AB*.

(i) Draw perpendicular bisector of *AB* which meets the circle at *D* and *E*.

Thus, *DE* is the required locus.

(ii) Join *AD* and draw angle bisector of $\angle DAB$ which meets the circle at *C*.

Thus, *AC* is the required locus.

(iii) Length of side $CD = 5$ cm.

Solution 11.

(a) Given : Scale = 1 : 50

(i) Let the actual height of the building be h m.

$\therefore$ $\frac{0.8}{h} = \frac{1}{50}$

$\Rightarrow$ $h = 50 \times 0.8 = 40$ m **Ans.**

(ii) Let the floor area of the model be x m^2.

$\therefore$ $\frac{x}{20} = \left(\frac{1}{50}\right)^2$

$\Rightarrow$ $\frac{x}{20} = \frac{1}{2500}$

$\Rightarrow$ $x = \frac{20}{2500}$ m^2

$= 0.008$ m^2 or 80 cm^2 **Ans.**

(b) Given : Height of cylinder (h) = 28 cm

Diameter of cylinder = 6 cm

$\Rightarrow$ Radius of cylinder (r) = $\frac{6}{2} = 3$ cm

Also, height of cones (H) = 10.5 cm

And, diameter of cones = 6 cm

$\Rightarrow$ Radius of cones (R) = $\frac{6}{2} = 3$ cm

Now, volume of solid cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 3^2 \times 28$

$= \frac{22}{7} \times 9 \times 28$

$= 792$ cm^3

And, volume of two cones

$= 2 \times \frac{1}{3}\pi R^2 H$

$= 2 \times \frac{1}{3} \times \frac{22}{7} \times 3^2 \times 10.5$

$= 198$ cm^3

So, volume of the remaining solid

$= (792 - 198)$ cm^3

$= 594$ cm^3 **Ans.**

(c) To prove :

$$\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2 = \tan^2\theta$$

Taking L.H.S. $= \left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2$

$= \left(\frac{1-\tan\theta}{1-\frac{1}{\tan\theta}}\right)^2$

$= \left(\frac{1-\tan\theta}{\frac{\tan\theta-1}{\tan\theta}}\right)^2$

$= \left(\frac{-\tan\theta(1-\tan\theta)}{1-\tan\theta}\right)^2$

$= (-\tan\theta)^2$

$= \tan^2\theta$

= R.H.S. **Hence Proved.**

MATHEMATICS 2019

QUESTIONS

SECTION—A (40 Marks)

*(Attempt **all** questions from this Section)*

Question 1.

(a) *Solve the following inequation and write down the solution set :* **[3]**

$$11x - 4 < 15x + 4 \le 13x + 14,\ x \in W$$

Represent the solution on a real number line.

(b) *A man invests ₹ 4500 in shares of a company which is paying 7.5% dividend. If ₹ 100 shares are available at a discount of 10%.* **[3]**

Find :

(i) *Number of shares he purchases.*

(ii) *His annual income.*

(c) *In a class of 40 students, marks obtained by the students in a class test (out of 10) are given below :* **[4]**

Marks	1	2	3	4	5	6	7	8	9	10
Number of Students	1	2	3	3	6	10	5	4	3	3

Calculate the following for the given distribution :

(i) *Median*

(ii) *Mode*

Question 2.

(a) *Using the factor theorem, show that (x– 2) is a factor of $x^3 + x^2 - 4x - 4$. Hence factorise the polynomial completely.* **[3]**

(b) *Prove that :* **[3]**

$$(\text{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$$

(c) *In an Arithmetic Progression (A.P.) the fourth and sixth terms are 8 and 14 respectively. Find the :* **[4]**

(i) *first term*

(ii) *common difference*

(iii) *sum of the first 20 terms*

Question 3.

(a) *Simplify :* **[3]**

$$\sin A\begin{bmatrix}\sin A & -\cos A\\ \cos A & \sin A\end{bmatrix} + \cos A\begin{bmatrix}\cos A & \sin A\\ -\sin A & \cos A\end{bmatrix}$$

(b) *M and N are two points on the X-axis and Y-axis respectively. P(3, 2) divides the line segment MN in the ratio 2 : 3.* **[3]**

Find :

(i) *the coordinates of M and N*

(ii) *slope of the line MN.*

(c) *A solid metallic sphere of radius 6 cm is melted and made into a solid cylinder of height 32 cm. Find the :*

(i) *radius of the cylinder*

(ii) *curved surface area of the cylinder* **[4]**

(Take $\pi = 3.1$)

Question 4.

(a) *The following numbers, K + 3, K + 2, 3K – 7 and 2K – 3 are in proportion. Find K.* **[3]**

(b) *Solve for x the quadratic equation $x^2 - 4x - 8 = 0$.*

Give your answer correct to three significant figures. **[3]**

(c) *Use ruler and compass only for answering this question.*

Draw a circle of radius 4 cm. Mark the centre as O. Mark a point P outside the circle at a distance of 7 cm from the centre. Construct two tangents to the circle from the external point P.

Measure and write down the length of any one tangent. **[4]**

SECTION—B (40 Marks)

*(Attempt any **four** questions from this Section)*

Question 5.

(a) *There are 25 discs numbered 1 to 25. They are put in a closed box and shaken thoroughly. A disc is drawn at random from the box.* **[3]**

Find the probability that the number on the disc is :

(i) *an odd number*

(ii) *divisible by 2 and 3 both*

(iii) *a number less than 16.*

(b) *Rekha opened a recurring deposit account for 20 months. The rate of interest is 9% per annum and Rekha receives ₹ 441 as interest at the time of maturity.*

Find the amount Rekha deposited each month. **[3]**

(c) *Use a graph sheet for this question.*

Take 1 cm = 1 unit along both X- and Y-axis. **[4]**

(i) *Plot the following points :*

A(0, 5), B(3, 0), C(1, 0) and D(1, – 5)

(ii) *Reflect the points B, C and D on the Y axis and name them as B′, C′ and D′ respectively.*

(iii) *Write down the coordinates of B′, C′ and D′.*

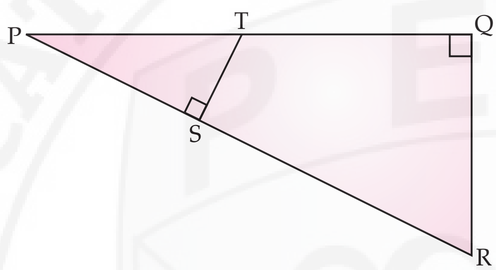
(iv) *Join the points A, B, C, D, D′, C′, B′, A in order and give a name to the closed figure ABCDD′C′B′.*

Question 6.

(a) *In the given figure, ∠PQR = ∠PST = 90°, PQ = 5 cm and PS = 2 cm.* **[3]**

(i) *Prove that ΔPQR ~ ΔPST.*

(ii) *Find-Area of ΔPQR : Area of quadrilateral SRQT.*



(b) *The first and last term of a Geometrical Progression (G.P.) are 3 and 96 respectively. If the common ratio is 2, find :* **[3]**

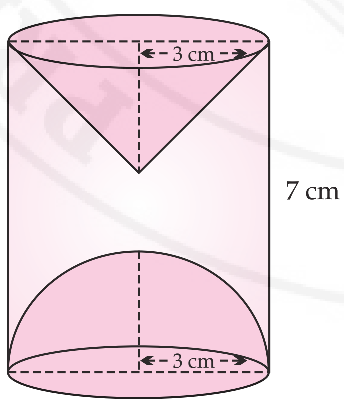
(i) *'n' the number of terms of the G.P.*

(ii) *Sum of the n terms.*

(c) *A hemispherical and a conical hole is scooped out of a solid wooden cylinder. Find the volume of the remaining solid where the measurements are as follows :*

The height of the solid cylinder is 7 cm, radius of each of hemisphere, cone and cylinder is 3 cm. Height of cone is 3 cm.

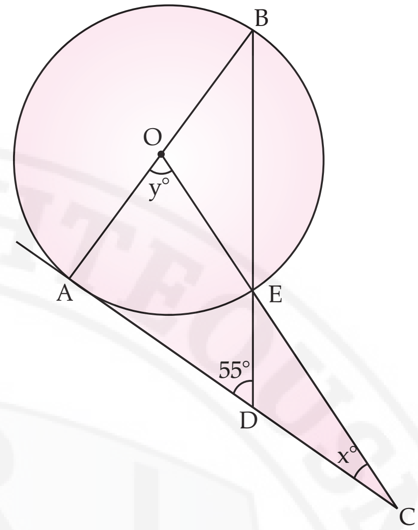
Give your answer correct to the nearest whole number. $\left(\text{Take } \pi = \frac{22}{7}\right)$ **[4]**



Question 7.

(a) *In the given figure AC is a tangent to the circle with centre O.*

If ∠ADB = 55°, find x and y. Give reasons for your answer. **[3]**



(b) *The model of a building is constructed with the scale factor 1 : 30.* **[3]**

(i) *If the height of the model is 80 cm, find the actual height of the building in meters.*

(ii) *If the actual volume of a tank at the top of the building is 27 m^3, find the volume of the tank on the top of the model.*

(c) *Given* $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} M = 6I$, *where M is a matrix and I is unit matrix of order 2 × 2.* **[4]**

(i) *State the order of matrix M.*

(ii) *Find the matrix M.*

Question 8.

(a) *The sum of the first three terms of an Arithmetic Progression (A.P.) is 42 and the product of the first and third term is 52. Find the first term and the common difference.* **[3]**

(b) *The vertices of a ΔABC are A (3, 8), B (– 1, 2) and C(6, – 6). Find :* **[3]**

(i) *Slope of BC.*

(ii) *Equation of a line perpendicular to BC and passing through A.*

(c) *Using ruler and a compass only construct a semi-circle with diameter BC = 7 cm. Locate a point A on the circumference of the semicircle such that A is equidistant from B and C. Complete the cyclic quadrilateral ABCD, such that D is equidistant from AB and BC. Measure ∠ADC and write it down.* **[4]**

Question 9.

(a) *The data on the number of patients attending a hospital in a month are given below. Find the average (mean) number of patients attending the hospital in a month by using the shortcut method.*

Take the assumed mean as 45. Give your answer correct to 2 decimal places. **[3]**

Number of patients	10-20	20-30	30-40	40-50	50-60	60-70
Number of Days	5	2	7	9	2	5

(b) *Using properties of proportion solve for x, given*

$$\frac{\sqrt{5x}+\sqrt{2x-6}}{\sqrt{5x}-\sqrt{2x-6}}=4$$ **[3]**

(c) *Sachin invests ₹ 8500 in 10% ₹ 100 shares at ₹ 170. He sells the shares when the price of each share rises by ₹ 30. He invests the proceeds in 12% ₹ 100 shares at ₹ 125. Find :* **[4]**

(i) *the sale proceeds.*

(ii) *the number of ₹ 125 shares he buys.*

(iii) *the change in his annual income.*

Question 10.

(a) *Use graph paper for this question.*

The marks obtained by 120 students in an English test are given below : **[6]**

Marks	Number of students
0 – 10	5
10 – 20	9
20 – 30	16
30 – 40	22
40 – 50	26
50 – 60	18
60 – 70	11
70 – 80	6
80 – 90	4
90 – 100	3

Draw the ogive and hence, estimate :

(i) *the median marks.*

(ii) *the number of students who did not pass test if the pass percentage was 50.*

(iii) *the upper quartile marks.*

(b) *A man observes the angle of elevation of the top of the tower to be 45°. He walks towards it in a horizontal line through its base. On covering 20 m the angle of elevation changes to 60°. Find the height of the tower correct to 2 significant figures.* **[4]**

Question 11.

(a) *Using the Remainder Theorem, find the remainders obtained when $x^3 + (kx + 8)x + k$ is divided by $x + 1$ and $x – 2$.*

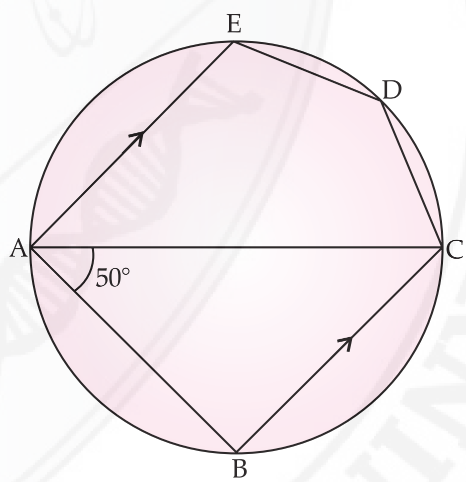
Hence find k if the sum of the two remainders is 1. **[3]**

(b) *The product of two consecutive natural numbers which are multiples of 3 is equal to 810. Find the two numbers.* **[3]**

(c) *In the given figure, ABCDE is a pentagon inscribed in a circle such that AC is a diameter and side BC || AE. If ∠BAC = 50°, find giving reasons :* **[4]**

(i) *∠ACB* **(ii)** *∠EDC* **(iii)** *∠BEC*

Hence, prove that BE is also a diameter.



ANSWERS

SECTION—A

Solution 1.

(a) Given, $11x – 4 < 15x + 4 \le 13x + 14, x \in W$.

$\therefore\ 11x – 4 < 15x + 4$ and $15x + 4 \le 13x + 14$

$\Rightarrow\ 11x – 15x < 4 + 4$ and $15x – 13x \le 14 – 4$

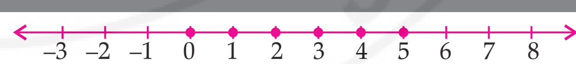
$\Rightarrow\ –4x < 8$ and $2x \le 10$

$\Rightarrow\ \frac{-4x}{-4} > \frac{8}{-4}$ and $\frac{2x}{2} \le \frac{10}{2}$

$\Rightarrow\ x > –2$ and $x \le 5$

$\therefore\ –2 < x \le 5$

$\therefore$ Solution set (W) = {0, 1, 2, 3, 4, 5}



(b) Given, Investment = ₹ 4500

Rate of dividend = 7.5%

Nominal value = ₹ 100, Discount = 10%

$\therefore$ Market value = ₹$\left(100-\frac{10}{100}\times 100\right)$ = ₹ 90

(i) Number of shares purchased

$$=\frac{\text{Investment}}{\text{Market Value}}$$

$$=\frac{4500}{90}=₹50$$ **Ans.**

(ii) Dividend per share = $\frac{7.5}{100} \times 100$ = ₹ 7.50

$\therefore$ Annual income = Dividend per share × Number of shares

= 7.50 × 50

= ₹ 375 **Ans.**

(c)

Marks	Number of Students	Cumulative Frequency
1	1	1
2	2	3
3	3	6
4	3	9
5	6	15
6	10	25
7	5	30
8	4	34
9	3	37
10	3	40
	$n = 40$	

Here, $n = 40$ (even)

(i) Median

$$= \frac{\frac{n}{2}^{\text{th}} \text{ observation} + \left(\frac{n}{2}+1\right)^{\text{th}} \text{ observation}}{2}$$

$$= \frac{20^{\text{th}} \text{ observation} + 21^{\text{st}} \text{ observation}}{2}$$

$$= \frac{6+6}{2} = 6$$ **Ans.**

(ii) $\because$ The highest frequency is 10.

$\therefore$ Mode = 6 **Ans.**

Solution 2.

(a) Let $f(x) = x^3 + x^2 - 4x - 4$

If $(x-2)$ is a factor of $f(x)$, then

$f(2) = 0$

Now, $f(2) = 2^3 + 2^2 - 4 \times 2 - 4$

$= 8 + 4 - 8 - 4$

$= 0$

$\therefore$ $(x-2)$ is a factor of $f(x)$.

$$\begin{array}{r|l} & x^2 + 3x + 2 \\ x-2 & x^3 + x^2 - 4x - 4 \\ & x^3 - 2x^2 \\ & - \quad + \\ \hline & 3x^2 - 4x \\ & 3x^2 + 6x \\ & - \quad + \\ \hline & 2x - 4 \\ & 2x - 4 \\ & - \quad + \\ \hline & \times \end{array}$$

Now, $x^2 + 3x + 2 = x^2 + 2x + x + 2$

$= x(x+2) + 1(x+2)$

$= (x+2)(x+1)$

$\therefore$ $f(x) = (x+1)(x+2)(x-2)$ **Ans.**

(b) To prove :

$(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$

L.H.S.= $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)$$

$$= \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta} \times \frac{1}{\sin\theta\cos\theta}$$

$$= \frac{\cos^2\theta}{\cos^2\theta} \times \frac{\sin^2\theta}{\sin^2\theta}$$

= 1 = R.H.S. **Hence Proved.**

(c) Let a and d be the first term and common difference of the given A.P. respectively

Then, $a_4 = 8$ and $a_6 = 14$

$\Rightarrow$ $a + 3d = 8$ ...(i)

and $a + 5d = 14$ ...(ii)

Subtracting equation (i) from (ii), we get

$2d = 6$

$\Rightarrow$ $d = 3$

Putting $d = 3$ in equation (i), we get

$a + 3 \times 3 = 8$

$\Rightarrow$ $a = 8 - 9 = -1$

(i) First term $(a) = -1$. **Ans.**

(ii) Common difference $(d) = 3$. **Ans.**

(iii) Sum of first 20 terms (S_{20})

$\because$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_{20} = \frac{20}{2}[2 \times (-1) + (20-1) \times 3]$

$= 10(-2 + 57)$

$= 550$ **Ans.**

Solution 3.

(a) $$\sin A \begin{bmatrix} \sin A & -\cos A \\ \cos A & \sin A \end{bmatrix} + \cos A \begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2 A & -\sin A\cos A \\ \sin A\cos A & \sin^2 A \end{bmatrix} + \begin{bmatrix} \cos^2 A & \sin A\cos A \\ -\sin A\cos A & \cos^2 A \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2 A + \cos^2 A & -\sin A\cos A + \sin A\cos A \\ \sin A\cos A - \sin A\cos A & \sin^2 A + \cos^2 A \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$[\because \sin^2 A + \cos^2 A = 1]$ **Ans.**

(b) Let $P(3, 2)$ divides the line segment joining $M(a, 0)$ and $N(0, b)$ in the ratio 2 : 3.

Here, $x = 3, x_1 = a, x_2 = 0, m_1 = 2$

$y = 2, y_1 = 0, y_2 = b, m_2 = 3$

Now, $x = \dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}$

$\Rightarrow \quad 3 = \dfrac{2 \times 0 + 3 \times a}{2 + 3}$

$\Rightarrow \quad 3 \times 5 = 0 + 3a$

$\Rightarrow \quad a = \dfrac{15}{3}$

$\Rightarrow \quad a = 5$

and $y = \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}$

$\Rightarrow \quad 2 = \dfrac{2 \times b + 3 \times 0}{2 + 3}$

$\Rightarrow \quad 2 \times 5 = 2b + 0$

$\Rightarrow \quad b = \dfrac{10}{2}$

$\Rightarrow \quad b = 5$

(i) The coordinates of $M = (a, 0) = (5, 0)$.

The coordinates of $N = (0, b) = (0, 5)$. **Ans.**

(ii) Slope of line $MN = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5 - 0}{0 - 5} = -1.$

Ans.

(c) Given, radius of sphere $(r_1) = 6$ cm and height of cylinder $(h) = 32$ cm

$\therefore$ Volume of sphere $(V_1) = \dfrac{4}{3}\pi r_1^3$

$= \dfrac{4}{3}\pi \times (6)^3 \text{ cm}^3$

Let radius of cylinder be r_2.

$\therefore$ Volume of cylinder, $(V_2) = \pi r_2^2 h$

$= \pi r_2^2 \times 32 \text{ cm}^3$

$\because \quad V_1 = V_2$

$\Rightarrow \quad \dfrac{4}{3}\pi \times 6^3 = \pi r_2^2 \times 32$

$\Rightarrow \quad r_2^2 = \dfrac{4 \times \pi \times 6^3}{3 \times \pi \times 32}$

$\Rightarrow \quad r_2^2 = 9$

$\Rightarrow \quad r_2 = 3$

(i) Radius of the cylinder, $(r_2) = 3$ cm **Ans.**

(ii) Curved surface area of the cylinder

$= 2\pi r_2 h$

$= 2 \times 3.1 \times 3 \times 32$

$= 595.2 \text{ cm}^2$ **Ans.**

Solution 4.

(a) Given, $K + 3$, $K + 2$, $3K - 7$ and $2K - 3$ are in proportion.

$\therefore \quad \dfrac{K+3}{K+2} = \dfrac{3K-7}{2K-3}$

$\Rightarrow \quad (K + 2)(3K - 7) = (K + 3)(2K - 3)$

$\Rightarrow \quad 3K^2 - 7K + 6K - 14 = 2K^2 - 3K + 6K - 9$

$\Rightarrow \quad 3K^2 - 2K^2 - K - 3K - 14 + 9 = 0$

$\Rightarrow \quad K^2 - 4K - 5 = 0$

$\Rightarrow \quad K^2 - 5K + K - 5 = 0$

$\Rightarrow \quad K(K - 5) + 1(K - 5) = 0$

$\Rightarrow \quad (K - 5)(K + 1) = 0$

$\Rightarrow \quad K - 5 = 0 \text{ or } K + 1 = 0$

$\therefore \quad K = 5 \text{ or } -1.$ **Ans.**

(b) Given quadratic equation is $x^2 - 4x - 8 = 0$.

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 1, b = -4$ and $c = -8$

$\therefore \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times (-8)}}{2 \times 1}$

$= \dfrac{4 \pm \sqrt{16 + 32}}{2} = \dfrac{4 \pm \sqrt{48}}{2} = \dfrac{4 \pm 6.928}{2}$

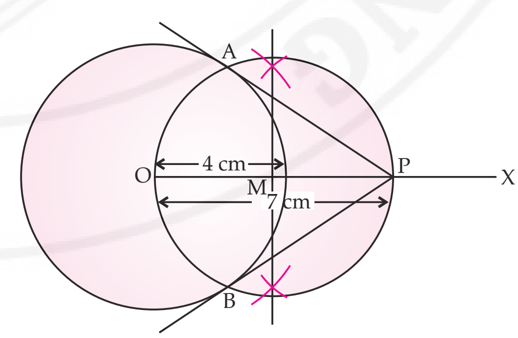
$= \dfrac{4 + 6.928}{2}$ or $\dfrac{4 - 6.928}{2}$

$= \dfrac{10.928}{2}$ or $\dfrac{-2.928}{2}$

$= 5.464$ or -1.464

$\therefore\ x = 5.464$ or -1.464 (correct to 3 significant figures). **Ans.**

(c) Given, radius = 4 cm and OP = 7 cm



Steps of constructions :

(i) Draw a circle of radius 4 cm with centre at O.

(ii) Draw a line OX and cut-off OP = 7 cm.

(iii) Bisect OP at M.

(iv) With M as centre, draw a circle passing through the points O and P to cut the previous circle at A and B.

(v) Join P with A and B. Hence, AP and BP are the required tangents.

$\therefore$ The length of tangent, $AP = 5.7$ cm **Ans.**

SECTION—B

Solution 5.

(a) Given, Total number of outcomes *i.e.*, $n(S) = 25$

(i) Let A be the event of getting an odd number.

$\therefore \quad A = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25\}$

$\therefore \quad n(A) = 13$

$\therefore \quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{13}{25}$ **Ans.**

(ii) Let B be the event of getting a number divisible by 2 and 3 both.

$\therefore \quad B = \{6, 12, 18, 24\}$

$\therefore \quad n(B) = 4$

$\therefore \quad P(B) = \dfrac{n(B)}{n(S)} = \dfrac{4}{25}$ **Ans.**

(iii) Let C be the event of getting a number less than 16.

$\therefore \quad C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

$\therefore \quad n(C) = 15$

$\therefore \quad P(C) = \dfrac{n(C)}{n(S)} = \dfrac{15}{25} = \dfrac{3}{5}$ **Ans.**

(b) Given, number of months $(n) = 20$

Rate of interest $(r) = 9\%$ p.a.

Interest received (I) = ₹ 441

Let the monthly deposit be ₹ P.

$$\therefore \quad I = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

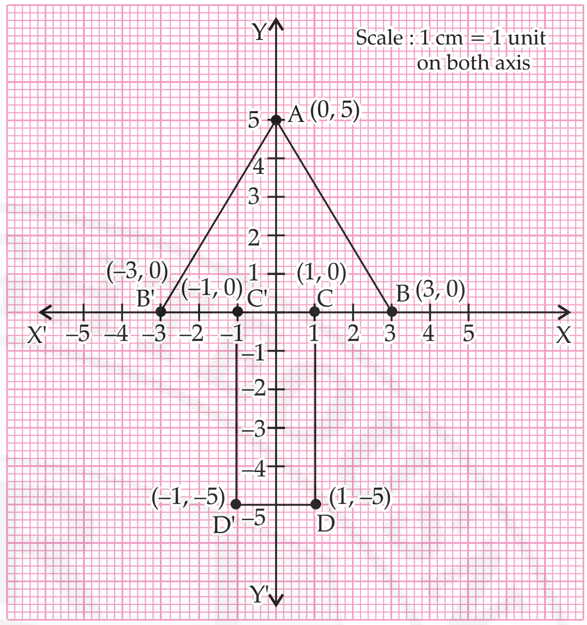
$$\Rightarrow \quad 441 = P \times \frac{20(20+1)}{2 \times 12} \times \frac{9}{100}$$

$$\Rightarrow \quad P = \frac{441 \times 2 \times 12 \times 100}{20 \times 21 \times 9} = ₹\ 280$$

$\therefore$ The required monthly deposit is ₹ 280. **Ans.**

(c) (i) The given points A (0, 5), B (3, 0), C (1, 0) and D (1, – 5) are plotted on the graph.

(ii) The points B, C and D are reflected on the Y-axis as B', C' and D' respectively.



(iii) The coordinates of

$B' = (-3, 0), \quad C' = (-1, 0),$

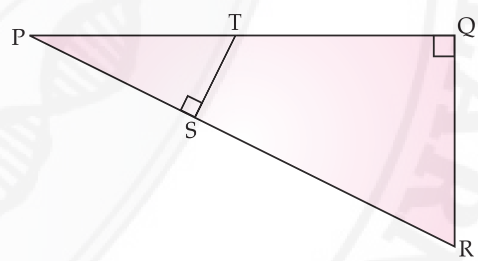
and $\quad D' = (-1, -5)$ **Ans.**

(iv) The name of the closed figure $ABCDD'C'B'$ is arrow or heptagon. **Ans.**

Solution 6.

(a) Given, $\quad \angle PQR = \angle PST = 90°$,

$PQ = 5$ cm and $PS = 2$ cm



(i) In ΔPQR and ΔPST,

$\angle PQR = \angle PST = 90°$ (Given)

$\angle QPR = \angle SPT$ (Common)

$\therefore \quad \Delta PQR \sim \Delta PST$ (By AA axiom)

Hence Proved.

(ii) $\dfrac{\text{Area of } \Delta PQR}{\text{Area of } \Delta PST} = \dfrac{PQ^2}{PS^2}$ $\quad (\because \Delta PQR \sim \Delta PST)$

$= \dfrac{5^2}{2^2} = \dfrac{25}{4}$

Now,

$$\frac{\text{Area of } \Delta PQR}{\text{Area of quadrilateral } SRQT} = \frac{\text{Area of } \Delta\ PQR}{\text{Area of } \Delta\ PQR - \text{Area of } \Delta PST}$$

$$= \frac{25\text{K}}{25\text{K} - 4\text{K}} = \frac{25}{21}$$ **Ans.**

(b) Given, first term $(a) = 3$, Last term $(a_n) = 96$ and common ratio $(r) = 2$.

(i) $\because \quad a_n = ar^{n-1}$

$\Rightarrow \quad 96 = 3 \times 2^{n-1}$

$\Rightarrow \quad \frac{96}{3} = 2^{n-1}$

$\Rightarrow \quad 32 = 2^{n-1}$

$\Rightarrow \quad 2^5 = 2^{n-1}$

$\Rightarrow \quad n - 1 = 5 \Rightarrow n = 5 + 1 \Rightarrow n = 6.$ **Ans.**

(ii) Sum of n terms $(S_n) = \frac{a(r^n - 1)}{r - 1}$

$= \frac{3(2^6 - 1)}{2 - 1}$

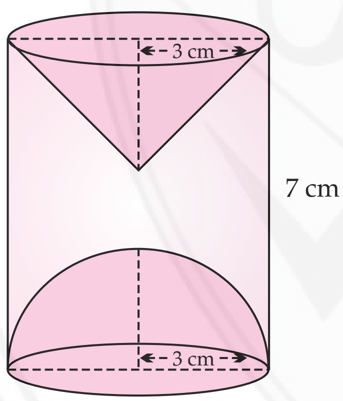
$= 3 \times 63$

$= 189$ **Ans.**

(c) Given, radius of each of hemisphere, cone and cylinder $(r) = 3$ cm.

Height of cylinder $(h_1) = 7$ cm

Height of cone $(h_2) = 3$ cm



The volume of the remaining solid

= Volume of cylinder – Volume of cone – Volume of hemisphere

$= \pi r^2 h_1 - \frac{1}{3}\pi r^2 h_2 - \frac{2}{3}\pi r^3$

$= \pi r^2 \left(h_1 - \frac{1}{3}h_2 - \frac{2}{3}r\right)$

$= \frac{22}{7} \times (3)^2 \left(7 - \frac{1}{3} \times 3 - \frac{2}{3} \times 3\right)$

$= \frac{22}{7} \times 9 \times 4$

$= 113.14 \approx 113 \text{ cm}^3$

(Correct to the nearest whole number) **Ans.**

Solution 7.

(a) Given, $\angle ADB = 55°$, AC is a tangent, $\angle ACO = x°$, $\angle AOE = y°$

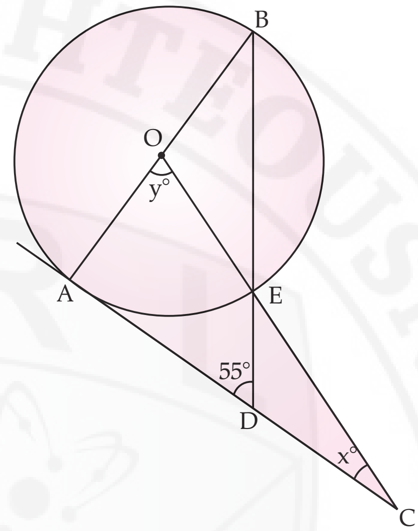
In ΔABD,

$\therefore \quad \angle BAD = 90°$ ($\because$ Radius OA is perpendicular to tangent AC)

and $\quad \angle ABD + \angle BAD + \angle ADB = 180°$ (Angle sum property)

$\Rightarrow \quad \angle ABD + 90° + 55° = 180°$

$\Rightarrow \quad \angle ABD = 180° - 145° = 35°$



$\therefore \quad \angle AOE = 2 \times \angle ABD$

(Angle at centre is twice the angle at circumference)

$\Rightarrow \quad y° = 2 \times 35°$

$\therefore \quad y° = 70°$

In ΔAOC,

$\angle ACO + \angle OAC + \angle AOC = 180°$

(Angle sum property)

$\Rightarrow \quad x° + 90° + 70° = 180°$ ($\angle OAC = 90°$, since radius is $\perp r$ to tangent)

$\Rightarrow \quad x° = 180° - 160°$

$= 20°$

Hence, $\quad x = 20°$

and $\quad y = 70°$ **Ans.**

(b) Given, scale factor, $1 : k = 1 : 30$ **Ans.**

(i) Actual height of the building $= k \times$ Height of the model

$= 30 \times 80$ cm

$= 2400$ cm

$= \frac{2400}{100}$ m

$= 24$ m **Ans.**

(ii) Actual volume of tank $= k^3 \times$ Volume of the model tank

$\Rightarrow \quad 27 \text{ m}^3 = (30)^3 \times$ Volume of the model tank

$\Rightarrow$ Volume of the model tank $= \dfrac{27\ m^3}{30 \times 30 \times 30}$

$= \dfrac{27 \times 100 \times 100 \times 100}{30 \times 30 \times 30} cm^3$

$= 1000\ cm^3$ **Ans.**

(c) Given, $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} M = 6I$

$\Rightarrow \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} M = 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} M = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$ ...(i)

(i) $(2 \times 2)\ (m \times n) = (2 \times 2) \rightarrow$ Order of matrix, $M = 2 \times 2$. **Ans.**

(ii) Let, $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$ [using (i)]

$\Rightarrow \begin{bmatrix} 4a+2c & 4b+2d \\ -a+c & -b+d \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

$\therefore$ $4a + 2c = 6$ ...(ii)

$-a + c = 0$ ...(iii) × 4

Solving equations (ii) and (iii),

$4a + 2c = 6$

$-4a + 4c = 0$

$6c = 6$

$\Rightarrow c = 1$

From equation (iii),

$-a + 1 = 0$

$\Rightarrow a = 1$

and $4b + 2d = 0$ ...(iv)

$\Rightarrow -b + d = 6$ ...(v) × 4

Solving equations (iv) and (v),

$4b + 2d = 0$

$-4b + 4d = 24$

$6d = 24$

$\Rightarrow d = 4$

From equation (iv),

$-b + 4 = 6$

$\Rightarrow -b = 2$

$\Rightarrow b = -2$

$\therefore M = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ **Ans.**

Solution 8.

(a) Let a and d be the first term and common difference respectively.

By first condition,

$a_1 + a_2 + a_3 = 42$

$\Rightarrow a + a + d + a + 2d = 42$

$\Rightarrow 3a + 3d = 42$

$\Rightarrow 3\ (a + d) = 42$

$\Rightarrow a + d = \dfrac{42}{3} = 14$

$\Rightarrow d = 14 - a$ ...(i)

By second condition,

$a_1 \times a_3 = 52$

$\Rightarrow a \times (a + 2d) = 52$

$\Rightarrow a^2 + 2ad = 52$ ...(ii)

From equations (i) and (ii), we have

$a^2 + 2a(14 - a) = 52$

$\Rightarrow a^2 + 28a - 2a^2 = 52$

$\Rightarrow -a^2 + 28a = 52$

$\Rightarrow a^2 - 28a + 52 = 0$

$\Rightarrow a^2 - 26a - 2a + 52 = 0$

$\Rightarrow a(a - 26) - 2(a - 26) = 0$

$\Rightarrow (a - 26)\ (a - 2) = 0$

$\Rightarrow a - 26 = 0$ or $a - 2 = 0$

$\Rightarrow a = 26$ or $a = 2$

$\therefore a = 26$ or 2

From equation (i),

when $a = 26$, $d = 14 - 26 = -12$

and when $a = 2$, $d = 14 - 2 = 12$ **Ans.**

Then, $d = 12$ or -12

(b) Given, $A\ (3, 8)$, $B\ (-1, 2)$ and $C(6, -6)$

(i) Slope of $BC\ (m_1) = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-6-2}{6-(-1)}$

$= \dfrac{-8}{7}$ **Ans.**

(ii) Slope of a line perpendicular to $BC\ (m)$

$= -\dfrac{1}{m_1}$

$= -\dfrac{1}{-8/7} = \dfrac{7}{8}$

Let the equation of the line perpendicular to BC and through A be

$y - y_1 = m\ (x - x_1)$

$\Rightarrow y - 8 = \dfrac{7}{8}\ (x - 3)$

$\Rightarrow 8\ (y - 8) = 7\ (x - 3)$

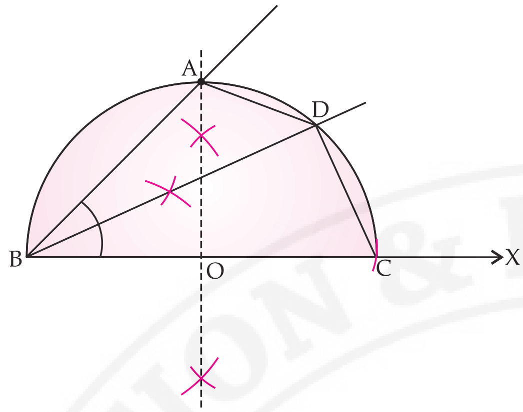
$\Rightarrow 8y - 64 = 7x - 21$

$\Rightarrow 7x - 8y - 21 + 64 = 0$

$\Rightarrow 7x - 8y + 43 = 0$

which is the required equation. **Ans.**

(c) Given, BC = 7 cm



Steps of construction :

1. Draw a line BX and cut off $BC = 7$ cm.
2. Bisect BC at O and draw a semicircle with centre at O and passing through B and C
3. Draw a perpendicular bisector of BC at O which intersects the semicircle at A.
4. Join AB and bisect $\angle ABC$ and the bisector cuts the semicircle at D.
5. Complete the cyclic quadrilateral $ABCD$ such that A is equidistant from B & C and D is equidistant from AB and BC.

$\therefore \angle ADC = 135°$ **Ans.**

Solution 9.

(a) Given, assumed mean (A) = 45.

Number of patients	Mid-value (x_i)	$d_i = x_i - A$	Number of days (f_i)	$f_i d_i$
10 - 20	15	– 30	5	– 150
20 - 30	25	– 20	2	– 40
30 - 40	35	– 10	7	– 70
40 - 50	45	0	9	0
50 - 60	55	10	2	20
60 - 70	65	20	5	100
			$\Sigma f_i = 30$	$\Sigma f_i d_i = -140$

$$\text{Mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= 45 + \left(-\frac{140}{30}\right)$$

$$= 45 - 4.667$$

$$= 40.333$$

$$= 40.33$$

(Correct to 2 decimal places) **Ans.**

(b) Given, $\dfrac{\sqrt{5x} + \sqrt{2x-6}}{\sqrt{5x} - \sqrt{2x-6}} = \dfrac{4}{1}$

Applying componendo and dividendo,

$$\frac{(\sqrt{5x} + \sqrt{2x-6}) + (\sqrt{5x} - \sqrt{2x-6})}{(\sqrt{5x} + \sqrt{2x-6}) - (\sqrt{5x} - \sqrt{2x-6})} = \frac{4+1}{4-1}$$

$$\Rightarrow \frac{\sqrt{5x} + \sqrt{2x-6} + \sqrt{5x} - \sqrt{2x-6}}{\sqrt{5x} + \sqrt{2x-6} - \sqrt{5x} + \sqrt{2x-6}} = \frac{5}{3}$$

$$\Rightarrow \frac{2\sqrt{5x}}{2\sqrt{2x-6}} = \frac{5}{3}$$

$$\Rightarrow \frac{\sqrt{5}}{\sqrt{2x-6}} = \frac{5}{3}$$

Squaring both sides, we get

$$\frac{5x}{2x-6} = \frac{25}{9}$$

$$\Rightarrow 25(2x-6) = 9 \times 5x$$

$$\Rightarrow 50x - 150 = 45x$$

$$\Rightarrow 50x - 45x = 150$$

$$\Rightarrow 5x = 150$$

$$\Rightarrow x = 30$$

Ans.

(c) Given, Investment = ₹ 8500,

Rate of dividend = 10%

Nominal Value = ₹ 100,

Market Value = ₹ 170

$$\therefore \text{Number of shares} = \frac{\text{Investment}}{\text{Market Value}}$$

$$= \frac{8500}{170} = 50$$

(i) Since the price of each share rises by ₹ 30,

Market Value of shares sold

= ₹ 170 + ₹ 30 = ₹ 200

$\therefore$ Sale proceeds = ₹ 200 × 50 = ₹ 10,000

Ans.

(ii) For new shares bought,

Investment = ₹ 10,000

Rate of dividend = 12%,

Nominal Value = ₹ 100,

and Market Value = ₹ 125.

$\therefore$ Number of shares bought

$$= \frac{\text{Investment}}{\text{Market Value}}$$

$$= \frac{10,000}{125}$$

= ₹ 80 **Ans.**

(iii) Annual income from old shares

= Dividend per share × Number of shares

$$= \frac{10}{100} \times 100 \times 50$$
$$= ₹\,500$$

Annual income from new shares

$$= \frac{12}{100} \times 100 \times 80$$
$$= ₹\,960$$

$\therefore$ The change in his annual income

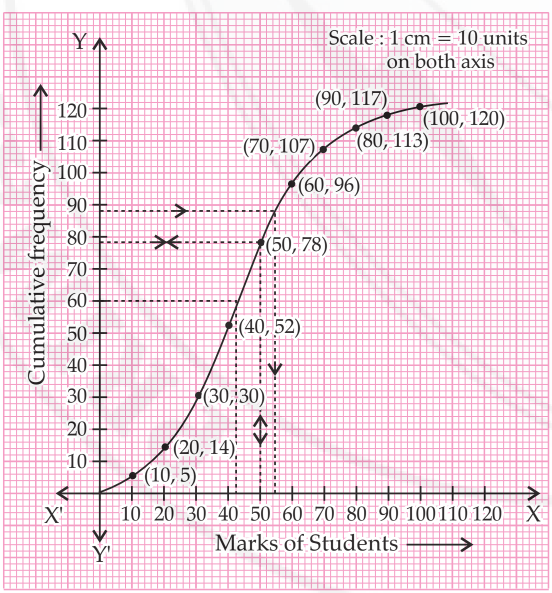
$$= ₹\,960 - ₹\,500$$
$$= ₹\,460 \qquad \textbf{Ans.}$$

Solution 10.

(a)

Marks	Number of students	Cumulative frequency
0 – 10	5	5
10 – 20	9	14
20 – 30	16	30
30 – 40	22	52
40 – 50	26	78
50 – 60	18	96
60 – 70	11	107
70 – 80	6	113
80 – 90	4	117
90 – 100	3	120

$\therefore \qquad N = 120$



(i) Median marks $= \frac{N}{2}$ th observation

$= \frac{120}{2}$ th observation

= 60th observation

= 43 (from ogive) **Ans.**

(ii) Number of students who did not pass

= 78 (from ogive) **Ans.**

(iii) Upper quartile $= \frac{3N}{4}$ th observation

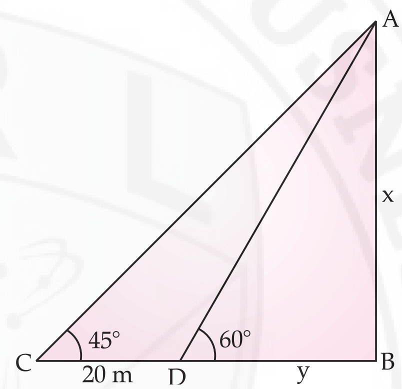
$= \frac{3 \times 120}{4}$ th observation

= 90th observation

= 56 (from ogive) **Ans.**

(b) Let $AB = x$ be the height of the tower and $CD = 20$ m be the distance he walked towards the tower

Let $\qquad BD = y$



In ΔABD,

$$\tan 60° = \frac{x}{y}$$
$$\Rightarrow \quad \sqrt{3} = \frac{x}{y}$$
$$\Rightarrow \quad y = \frac{x}{\sqrt{3}} \qquad ...(i)$$

In ΔABC,

$$\tan 45° = \frac{x}{y+20}$$
$$\Rightarrow \quad 1 = \frac{x}{y+20}$$
$$\Rightarrow \quad x = y + 20 \qquad ...(ii)$$

From equations (i) and (ii), we get

$$x = \frac{x}{\sqrt{3}} + 20$$
$$\Rightarrow \quad \sqrt{3}x = x + 20\sqrt{3}$$
$$\Rightarrow \quad \sqrt{3}x - x = 20\sqrt{3}$$
$$\Rightarrow \quad (\sqrt{3} - 1)\,x = 20\sqrt{3}$$
$$\Rightarrow \quad x = \frac{20\sqrt{3}}{\sqrt{3} - 1}$$
$$\Rightarrow \quad x = \frac{20\sqrt{3}\,(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$
$$\Rightarrow \quad x = \frac{20\sqrt{3}\,(\sqrt{3}+1)}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{20\sqrt{3}\,(\sqrt{3}+1)}{3-1}$$
$$= \frac{20\sqrt{3}\,(\sqrt{3}+1)}{2}$$
$$= 10\sqrt{3}\,(\sqrt{3}+1)$$
$$= 10\sqrt{3}\times\sqrt{3}+10\sqrt{3}$$
$$= 30 + 10 \times 1.732$$
$$= 30 + 17.32$$
$$= 47.32 \text{ m}$$

(Correct to 2 significant figures)

∴ Height of tower is 47.32 m. **Ans.**

Solution 11.

(a) Let $f(x) = x^3 + (kx+8)\,x + k$

when $f(x)$ is divided by $(x+1)$ then by remainder theorem

Remainder, $f(-1) = (-1)^3 + \{k(-1)+8\}(-1) + k$
$= -1 + (-k+8)(-1) + k$
$= -1 + k - 8 + k$
$= 2k - 9$

when $f(x)$ is divided by $(x-2)$,

Remainder, $f(2) = (2)^3 + (k.2+8)\,2 + k$
$= 8 + 4k + 16 + k$
$= 5k + 24$

Also, sum of remainders = 1

$f(-1) + f(2) = 1$

$\Rightarrow \quad 2k - 9 + 5k + 24 = 1$

$\Rightarrow \quad 7k + 15 = 1$

$\Rightarrow \quad 7k = 1 - 15$

$\Rightarrow \quad k = \frac{-14}{7} = -2$ **Ans.**

(b) Let the two consecutive natural numbers which are multiples of 3 be x and $x+3$.

According to the question,

$x(x+3) = 810$

$\Rightarrow \quad x^2 + 3x = 810$

$\Rightarrow \quad x^2 + 3x - 810 = 0$

$\Rightarrow \quad x^2 + 30x - 27x - 810 = 0$

$\Rightarrow \quad x(x+30) - 27(x+30) = 0$

$\Rightarrow \quad (x+30)(x-27) = 0$

$\Rightarrow \quad x + 30 = 0$ or $x - 27 = 0$

$\Rightarrow \quad x = -30$ or $x = 27$

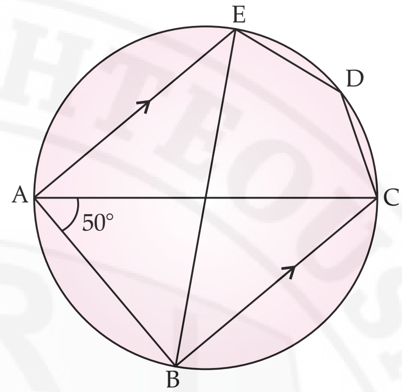
∴ $x = 27$

(∵ – 30 is not a natural number)

∴ $x + 3 = 27 + 3 = 30$

Hence, the two numbers are 27 and 30. **Ans.**

(c) Given, AC is diameter, $BC \,||\, AE$, and $\angle BAC = 50°$



(i) $\angle ABC = 90°$

(∵ *Angle at circumference of a semicircle*)

In ΔABC,

∴ $\angle ACB + \angle BAC + \angle ABC = 180°$

(Angles sum property)

$\Rightarrow \quad \angle ACB + 50° + 90° = 180°$

$\Rightarrow \quad \angle ACB = 180° - 140°$

$\angle ACB = 40°$ **Ans.**

(ii) $\angle CAE = \angle ACB$

(Alternate angles as $BC \,||\, AE$)

$= 40°$

∴ $\angle EDC + \angle CAE = 180°$

(Sum of opposite angles of a cyclic quadrilateral is 180°)

$\Rightarrow \quad \angle EDC + 40° = 180°$

$\Rightarrow \quad \angle EDC = 180° - 40°$

$\angle EDC = 140°$ **Ans.**

(iii) $\angle BEC = \angle BAC$

(Angles on same segment are equal)

$= 50°$ **Ans.**

Now, $\angle BAE = \angle BAC + \angle CAE$
$= 50° + 40°$
$= 90°$

We know that, if an angle of a triangle in a circle is 90°. Then, the hypotenuse must be the diameter of the circle.

Hence, BE is a diameter (∵ $\angle BAE = 90°$)

Hence Proved.

MATHEMATICS 2018

QUESTIONS

SECTION—A (40 Marks)

*(Attempt **all** questions from this Section)*

Question 1.

(a) *Find the value of 'x' and 'y' if :* **[3]**

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

(b) *Sonia had recurring deposit account in a bank and deposited ₹ 600 per month for $2\frac{1}{2}$ years. If the rate of interest was 10% p.a., find the maturity value of this account.* **[3]**

(c) *Cards bearing numbers 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card which is:* **[4]**

(i) *a prime number.*

(ii) *a number divisible by 4.*

(iii) *a number that is a multiple of 6.*

(iv) *an odd number.*

Question 2.

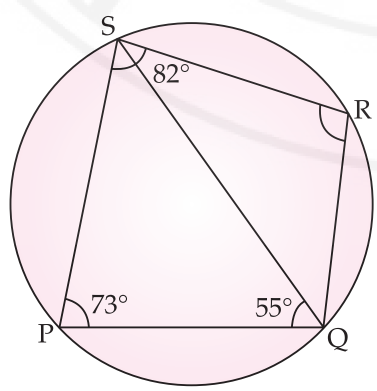
(a) *The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. Find the* **[3]**

(i) *radius of the cylinder*

(ii) *volume of cylinder.* $\left(\text{use } \pi = \frac{22}{7}\right)$

(b) *If (k – 3), (2k + 1) and (4k + 3) are three consecutive terms of an A.P., find the value of k.* **[3]**

(c) *PQRS is a cyclic quadrilateral. Given, ∠QPS = 73°, ∠PQS = 55° and ∠PSR = 82°, calculate :* **[4]**



(i) *∠QRS*

(ii) *∠RQS*

(iii) *∠PRQ*

Question 3.

(a) *If (x + 2) and (x + 3) are factors of $x^3 + ax + b$, find the values of 'a' and 'b'.* **[3]**

(b) *Prove that* $\sqrt{\sec^2\theta + \text{cosec}^2\theta} = \tan\theta + \cot\theta$ **[3]**

(c) *Using a graph paper draw a histogram for the given distribution showing the number of runs scored by 50 batsmen. Estimate the mode of the data :* **[4]**

Runs scored	3000–4000	4000–5000	5000–6000	6000–7000	7000–8000	8000–9000	9000–10000
No. of bats-men	4	18	9	6	7	2	4

Question 4.

(a) *Solve the following inequation, write down the solution set and represent it on the real number line :* **[3]**

$$-2 + 10x \le 13x + 10 < 24 + 10x, x \in Z$$

(b) *If the straight lines 3x – 5y = 7 and 4x + ay + 9 = 0 are perpendicular to one another, find the value of a.* **[3]**

(c) *Solve $x^2 + 7x = 7$ and give your answer correct to two decimal places.* **[4]**

SECTION—B (40 Marks)

*(Attempt any **four** questions from this Section)*

Question 5.

(a) *The 4th term of a G.P. is 16 and the 7th term is 128. Find the first term and common ratio of the series.* **[3]**

(b) *A man invests ₹ 22,500 in ₹ 50 shares available at 10% discount. If the dividend paid by the company is 12%, calculate :* **[3]**

(i) *The number of shares purchased*

(ii) *The annual dividend received*

(iii) *The rate of return he gets on his investment. Give your answer correct to the nearest whole number.*

(c) *Use graph paper for this question (Take 2 cm = 1 unit along both X and Y axis). ABCD is a quadrilateral whose vertices are A(2, 2), B (2, – 2), C(0, – 1) and D(0, 1).* **[4]**

(i) *Reflect quadrilateral ABCD on the Y-axis and name it as A'B'CD.*

(ii) *Write down the coordinates of A' and B'.*

(iii) *Name two points which are invariant under the above reflection.*

(iv) *Name the polygon A'B'CD.*

Question 6.

(a) *Using properties of proportion, solve for x. Given that x is positive :* **[3]**

$$\frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = 4$$

(b) *If* $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$ *and* $C = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$, *find* $AC + B^2 - 10C$. **[3]**

(c) *Prove that* $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$ **[4]**

Question 7.

(a) *Find the value of k for which the following equation has equal roots :* **[3]**

$$x^2 + 4kx + (k^2 - k + 2) = 0$$

(b) *On a map drawn to a scale of 1 : 50,000, a rectangular plot of land ABCD has the following dimensions. AB = 6 cm; BC = 8 cm and all angles are right angles. Find:* **[3]**

(i) *the actual length of the diagonal distance AC of the plot in km.*

(ii) *the actual area of the plot in sq. km.*

(c) *A(2, 5), B(– 1, 2) and C(5, 8) are the vertices of a triangle ABC, 'M' is a point on AB such that AM : MB = 1 : 2. Find the coordinates of 'M'. Hence, find the equation of the line passing through the points C and M.* **[4]**

Question 8.

(a) *₹ 7500 were divided equally among a certain number of children. Had there been 20 less children, each would have received ₹ 100 more. Find the original number of children.* **[3]**

(b) *If the mean of the following distribution is 24, find the value of 'a'.* **[3]**

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Number of students	7	a	8	10	5

(c) *Using ruler and compass only, construct a ΔABC such that BC = 5 cm and AB = 6.5 cm and ∠ABC = 120°.* **[4]**

(i) *Construct a circumcircle of ΔABC*

(ii) *Construct a cyclic quadrilateral ABCD, such that D is equidistant from AB and BC.*

Question 9.

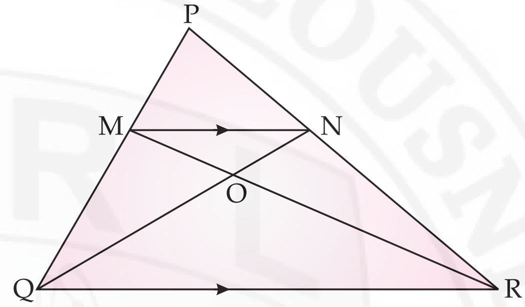
(a) *Priyanka has a recurring deposit account of ₹ 1000 per month at 10% per annum. If she gets ₹ 5550 as interest at the time of maturity, find the total time for which the account was held.* **[3]**

(b) *In ΔPQR, MN is parallel to QR and* $\frac{PM}{MQ} = \frac{2}{3}$ **[3]**

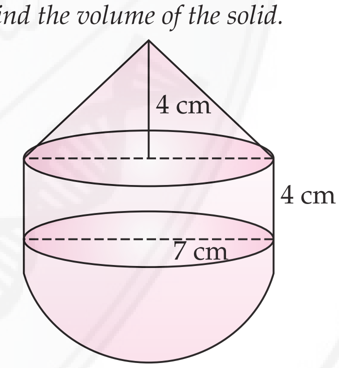
(i) *Find* $\frac{MN}{QR}$

(ii) *Prove that ΔOMN and ΔORQ are similar.*

(iii) *Find, Area of ΔOMN : Area of ΔORQ.*



(c) *The following figure represents a solid consisting of right circular cylinder with a hemisphere at one end and a cone at the other. Their common radius is 7 cm. The height of the cylinder and cone are each of 4 cm. Find the volume of the solid.* **[4]**

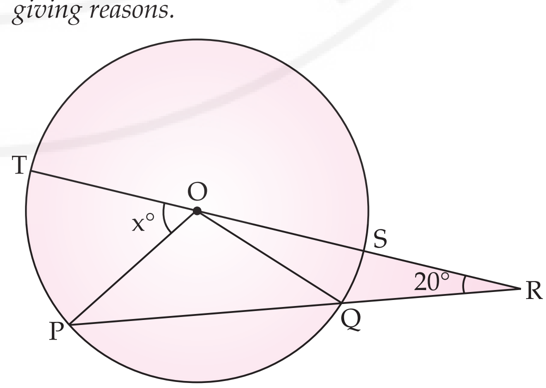


Question 10.

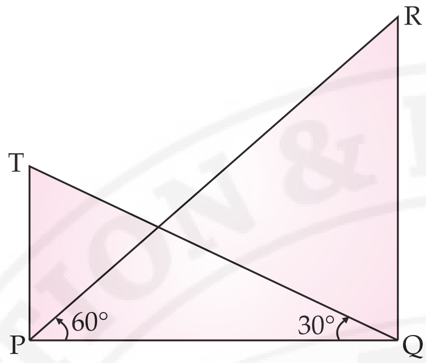
(a) *Use remainder theorem to factorize the following polynomial :* **[3]**

$$2x^3 + 3x^2 - 9x - 10.$$

(b) *In the figure given below 'O' is the centre of the circle. If QR = OP and ∠ ORP = 20°. Find the value of 'x' giving reasons.* **[3]**



(c) *The angle of elevation from a point P of the top of a tower QR, 50 m high is 60° and that of the tower PT from a point Q is 30°. Find the height of the tower PT, correct to the nearest metre.* **[4]**



Question 11.

(a) *The 4th term of an A.P. is 22 and 15th term is 66. Find the first term and the common difference. Hence, find the sum of the series to 8 terms.* **[4]**

(b) *Use Graph paper for this question.* **[6]**

A survey regarding height (in cm) of 60 boys belonging to Class 10 of a school was conducted. The following data was recorded :

Height in cm	135–140	140–145	145–150	150–155	155–160	160–165	165–170
No. of boys	4	8	20	14	7	6	1

Taking 2 cm = height of 10 cm along one axis and 2 cm =10 boys along the other axis draw an ogive of the above distribution. Use the graph to estimate the following :

(i) *the median*

(ii) *lower quartile*

(iii) *if above 158 cm is considered as the tall boys of the class. Find the number of boys in the class who are tall.*

ANSWERS

SECTION—A

Solution 1.

(a) We have,

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 14 \\ 18 & 2y-10 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+6 & 7 \\ 22 & 2y-5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

On comparing both sides, we get

$$\Rightarrow \quad 2x + 6 = 10, \qquad 2y - 5 = 15$$
$$\Rightarrow \quad 2x = 10 - 6, \qquad 2y = 15 + 5$$
$$\Rightarrow \quad 2x = 4, \qquad 2y = 20$$
$$\Rightarrow \quad x = \frac{4}{2}, \qquad y = \frac{20}{2}$$
$$\therefore \quad x = 2, y = 10.$$ **Ans.**

(b) Here, P = ₹ 600, $n = 2\frac{1}{2}$ years = 30 months, $r = 10\%$

$$\therefore \quad \text{Interest, I} = \text{P} \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$
$$= 600 \times \frac{30 \times 31}{2 \times 12} \times \frac{10}{100}$$
$$= ₹\ 2325$$

∴ Maturity value,

M.V. = Pn + I

= 600 × 30 + 2325

= ₹ 20325. **Ans.**

(c) Here, Sample Space,

S = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}

∴ n(S) = 10

(i) Let A be the event of getting a prime number.

A = {2}

∴ n(A) = 1

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{1}{10}$$ **Ans.**

(ii) Let B be the event of getting a number divisible by 4.

∴ B = {4, 8, 12, 16, 20}

∴ n(B) = 5

$$\therefore \quad P(B) = \frac{n(B)}{n(S)} = \frac{5}{10} = \frac{1}{2}$$ **Ans.**

(iii) Let C be the event of getting a number which is multiple of 6.

∴ C = {6, 12, 18}

∴ n(C) = 3

$$\therefore \quad P(C) = \frac{n(C)}{n(S)} = \frac{3}{10}$$ **Ans.**

(iv) Let D be the event of getting an odd number.

∴ D = { }

∴ n(D) = 0

$$\therefore \quad P(D) = \frac{n(D)}{n(S)} = \frac{0}{10} = 0$$ **Ans.**

Solution 2.

(a) Given, circumference of base of cylinder = 132 cm, height of cylinder, h = 25 cm.

(i) Let r be the radius of cylinder

$\therefore \quad 2\pi r = 132$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 132$

$\Rightarrow \quad r = \frac{132 \times 7}{2 \times 22} = 21$ cm **Ans.**

(ii) Volume of the cylinder

$= \pi r^2 h$

$= \frac{22}{7} \times (21)^2 \times 25$

$= 34650 \text{ cm}^3$. **Ans.**

(b) Given, $(k - 3)$, $(2k + 1)$, $(4k + 3)$ are 3 consecutive terms of an A.P.

As the difference between the consecutive terms in A.P. are same, *i.e.*, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = d$.

$\therefore \quad (2k + 1) - (k - 3) = (4k + 3) - (2k + 1)$

$\Rightarrow \quad 2k + 1 - k + 3 = 4k + 3 - 2k - 1$

$\Rightarrow \quad k + 4 = 2k + 2$

$\Rightarrow \quad k - 2k = 2 - 4$

$\Rightarrow \quad -k = -2$

$\Rightarrow \quad k = 2$ **Ans.**

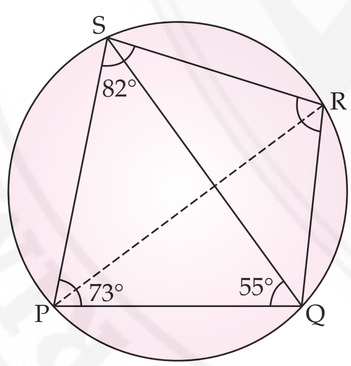
(c) Given, $\angle QPS = 73°$, $\angle PQS = 55°$, $\angle PSR = 82°$

(i) $\angle QRS + \angle QPS = 180°$

(opposite angles of a cyclic quadrilateral are supplementary)

$\Rightarrow \quad \angle QRS + 73° = 180°$

$\Rightarrow \quad \angle QRS = 180° - 73° = 107°$. **Ans.**



(ii) $\angle PQR + \angle PSR = 180°$

(opposite angles of a cyclic quadrilateral are supplementary)

$\Rightarrow \quad \angle PQR + 82° = 180°$

$\Rightarrow \quad \angle PQR = 180° - 82°$

$\Rightarrow \quad \angle PQR = 98°$

$\Rightarrow \quad \angle RQS + \angle PQS = 98°$

$\Rightarrow \quad \angle RQS + 55° = 98°$

$\Rightarrow \quad \angle RQS = 98° - 55° = 43°$. **Ans.**

(iii) $\angle PSQ + \angle QPS + \angle PQS = 180°$

(sum of angles of a triangle is 180°)

$\Rightarrow \quad \angle PSQ + 73° + 55° = 180°$

$\Rightarrow \quad \angle PSQ = 180° - 128° = 52°$

$\therefore \quad \angle PRQ = \angle PSQ$

(angles on same segment are equal)

$\Rightarrow \quad \angle PRQ = 52°$. **Ans.**

Solution 3.

(a) Let $f(x) = x^3 + ax + b$

$\because (x + 2)$ and $(x + 3)$ are factors of $f(x)$

$\therefore \quad f(-2) = 0$

$\Rightarrow \quad (-2)^3 + a(-2) + b = 0$

$\Rightarrow \quad -8 - 2a + b = 0$

$\Rightarrow \quad -2a + b = 8$ ...(i)

Also, $f(-3) = 0$

$\Rightarrow \quad (-3)^3 + a(-3) + b = 0$

$\Rightarrow \quad -27 - 3a + b = 0$

$\Rightarrow \quad -3a + b = 27$ ...(ii)

Subtracting equation (ii) from equation (i), we have

$$\begin{array}{r} -2a + b = 8 \\ -3a + b = 27 \\ + \quad - \quad - \\ \hline a = -19 \end{array}$$

Putting the value of a in equation (i)

$-2 \times (-19) + b = 8$

$\Rightarrow \quad b = 8 - 38 = -30$

$\therefore \quad a = -19, b = -30$. **Ans.**

(b) To prove,

$\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta$

$\therefore \quad \text{L.H.S.} = \sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}$

$= \sqrt{1 + \tan^2\theta + 1 + \cot^2\theta}$

$[\because 1 + \tan^2\theta = \sec^2\theta, 1 + \cot^2\theta = \operatorname{cosec}^2\theta]$

$= \sqrt{\tan^2\theta + \cot^2\theta + 2}$

$= \sqrt{\tan^2\theta + \cot^2\theta + 2\tan\theta.\cot\theta}$

$[\because \tan\theta.\cot\theta = 1]$

$= \sqrt{(\tan\theta + \cot\theta)^2}$

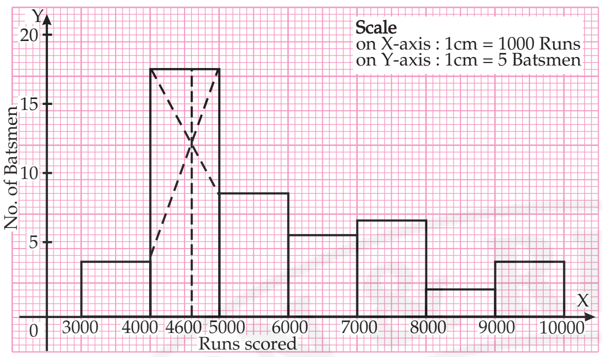
$[\because (a + b)^2 = a^2 + b^2 + 2ab]$

$= \tan\theta + \cot\theta$

$=$ R.H.S. **Hence Proved.**

(c)

Runs Scored	No. of batsmen
3000 – 4000	4
4000 – 5000	18
5000 – 6000	9
6000 – 7000	6
7000 – 8000	7
8000 – 9000	2
9000 – 10000	4



$\therefore$ Mode = 4600 **Ans.**

Solution 4.

(a) Given inequation is,

$$-2 + 10x \le 13x + 10 < 24 + 10x, x \in Z$$

$\Rightarrow$ $-2 + 10x \le 13x + 10;$

$\Rightarrow$ $10x - 13x \le 10 + 2;$

$\Rightarrow$ $-3x \le 12;$

$\Rightarrow$ $-x \le 4;$

$\Rightarrow$ $x \ge -4;$

and $13x + 10 < 24 + 10x$

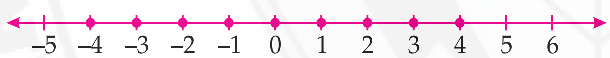
$\Rightarrow$ $13x - 10x < 24 - 10$

$\Rightarrow$ $3x < 14$

$\Rightarrow$ $x < \frac{14}{3}$

$\therefore$ $-4 \le x < 4\frac{2}{3}$

$\therefore$ Solution set = {– 4, – 3, – 2, – 1, 0, 1, 2, 3, 4}



Ans.

(b) Given equation of lines are $3x - 5y = 7$ and $4x + ay + 9 = 0$

$\Rightarrow$ $-5y = -3x + 7$ and $ay = -4x - 9$

$\Rightarrow$ $y = \frac{3}{5}x - \frac{7}{5}$ and $y = -\frac{4}{a}x - \frac{9}{a}$

Comparing both equations with $y = mx + c$, we get

$$m_1 = \frac{3}{5} \qquad m_2 = -\frac{4}{a}$$

The lines are perpendicular to each other,

$\therefore$ $m_1 \times m_2 = -1$

$\Rightarrow$ $\frac{3}{5} \times \left(-\frac{4}{a}\right) = -1$

$\Rightarrow$ $-\frac{12}{5} = -a$

$\Rightarrow$ $a = \frac{12}{5}$.

$\Rightarrow$ $a = 2\frac{2}{5}$. **Ans.**

(c) We have, $x^2 + 7x = 7$

$\Rightarrow$ $x^2 + 7x - 7 = 0$

Comparing it with $ax^2 + bx + c = 0$, we have

$a = 1, b = 7, c = -7$

$\therefore$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-7 \pm \sqrt{7^2 - 4 \times 1 \times (-7)}}{2 \times 1}$

$= \frac{-7 \pm \sqrt{49 + 28}}{2}$

$= \frac{-7 \pm \sqrt{77}}{2}$

$= \frac{-7 \pm 8.775}{2}$

$= \frac{-7 + 8.775}{2}$ or $\frac{-7 - 8.775}{2}$

$= \frac{1.775}{2}$ or $\frac{-15.775}{2}$

$= 0.8875$ or -7.8875

$= 0.89$ or -7.89

(correct to 2 decimal places)

Ans.

SECTION—B

Solution 5.

(a) Let a be the first term and r be the common ratio of the given G.P.

$\therefore$ $T_4 = 16$ and $T_7 = 128$

$\Rightarrow$ $ar^3 = 16$ ...(i)

and $ar^6 = 128$ ...(ii)

Dividing equation (ii) by equation (i), we get

$$\frac{ar^6}{ar^3} = \frac{128}{16}$$

$\Rightarrow$ $r^3 = 8$

$\Rightarrow$ $r = 2$

$\therefore$ From equation (i), $a \times 2^3 = 16$

$\Rightarrow$ $a = \frac{16}{8} = 2$

$\therefore$ $a = 2, r = 2.$ **Ans.**

(b) Given, investment = ₹ 22,500, N.V. = ₹ 50, discount = 10%

$\therefore$ M.V. = ₹ $\left(50 - \frac{10}{100} \times 50\right)$ = ₹ 45

Rate of dividend = 12%

(i) Number of shares = $\frac{\text{Investment}}{\text{M.V.}}$

$= \frac{22500}{45} = 500$ **Ans.**

(ii) Annual dividend = Dividend per share × No. of shares

$$= \frac{12}{100} \times 50 \times 500$$

$= ₹\ 3000$ **Ans.**

(iii) Rate of return $= \frac{\text{Dividend}}{\text{Investment}} \times 100\%$

$$= \frac{3000}{22500} \times 100\%$$

$= 13.3\% = 13\%$

(correct to the nearest whole number)

Ans.

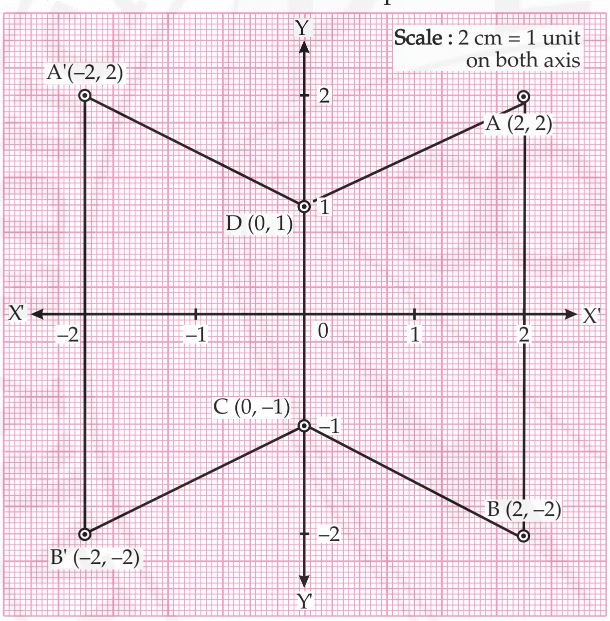
(c) **(i)** Reflected quadrilateral A′B′CD is shown in graph.

(ii) Coordinates of A′ = (– 2, 2)

Coordinates of B′ = (– 2, – 2)

(iii) Two invariant points are C(0, – 1) and D (0, 1)

(iv) A′B′CD is an isosceles trapezium.



Solution 6.

(a) Given, $\frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = \frac{4}{1}$

$\Rightarrow \frac{2x + \sqrt{4x^2 - 1} + 2x - \sqrt{4x^2 - 1}}{2x + \sqrt{4x^2 - 1} - 2x + \sqrt{4x^2 - 1}} = \frac{4 + 1}{4 - 1}$

(using componendo and dividendo)

$\Rightarrow \frac{4x}{2\sqrt{4x^2 - 1}} = \frac{5}{3}$

$\Rightarrow 10\sqrt{4x^2 - 1} = 12x$

$\Rightarrow 100(4x^2 - 1) = 144x^2$

(On squaring both sides)

$\Rightarrow 400\,x^2 - 100 = 144x^2$

$\Rightarrow 400x^2 - 144x^2 = 100$

$\Rightarrow 256x^2 = 100$

$\Rightarrow x^2 = \frac{100}{256}$

$\Rightarrow x^2 = \left(\frac{10}{16}\right)^2$

$\Rightarrow x = \pm\frac{10}{16} = \pm\frac{5}{8}$

$\therefore x = \frac{5}{8}$ ($\because x$ is positive)

Ans.

(b) Given, $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}, B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$

$\therefore AC + B^2 - 10\,C = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} - 10\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$

$= \begin{bmatrix} 2-3 & 0+12 \\ 5-7 & 0+28 \end{bmatrix} + \begin{bmatrix} 0-4 & 0+28 \\ 0-7 & -4+49 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$

$= \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix} + \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$

$= \begin{bmatrix} -5 & 40 \\ -9 & 73 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$

$= \begin{bmatrix} -15 & 40 \\ 1 & 33 \end{bmatrix}$ **Ans.**

(c) To prove, $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$

$\therefore$ L.H.S. $= (1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta)$

$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$

$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)$

$= \frac{(\sin\theta + \cos\theta)^2 - (1)^2}{\sin\theta\cos\theta}$

$[\because (a + b)(a - b) = a^2 - b^2]$

$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$

$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$

= 2 = R.H.S. **Hence Proved.**

Solution 7.

(a) Given equation is, $x^2 + 4kx + (k^2 - k + 2) = 0$

Comparing it with $ax^2 + bx + c = 0$, we have,

$$a = 1, b = 4k, c = k^2 - k + 2.$$

$$\therefore \quad D = b^2 - 4ac = (4k)^2 - 4 \times 1 \times (k^2 - k + 2)$$

$$= 16k^2 - 4k^2 + 4k - 8$$

$$= 12k^2 + 4k - 8$$

$\because$ The roots of given equation are equal, so

$$D = 0$$

$$\Rightarrow 12k^2 + 4k - 8 = 0$$

$$\Rightarrow 3k^2 + k - 2 = 0$$

$$\Rightarrow 3k^2 + 3k - 2k - 2 = 0$$

$$\Rightarrow 3k(k + 1) - 2(k + 1) = 0$$

$$\Rightarrow (k + 1)(3k - 2) = 0$$

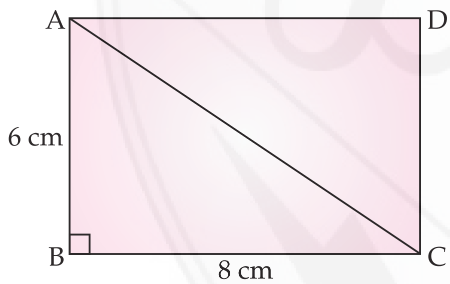
$$\Rightarrow k + 1 = 0 \text{ or } 3k - 2 = 0$$

$$\Rightarrow k = -1 \text{ or } k = \frac{2}{3}$$

$\therefore$ The value of k is -1 or $\frac{2}{3}$. **Ans.**

(b) Here, $1 : k = 1 : 50{,}000$

and AB = 6 cm, BC = 8 cm



$$\therefore \quad AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{6^2 + 8^2}$$

$$= \sqrt{36 + 64}$$

$$= \sqrt{100} = 10 \text{ cm}$$

(i) Actual length of AC = $k \times$ AC

$$= 50{,}000 \times 10 \text{ cm}$$

$$= 5{,}00{,}000 \text{ cm}$$

$$= \frac{500000}{100000} \text{ km} = 5 \text{ km.}$$

Ans.

(ii) Area of rectangle ABCD

$$= 6 \times 8 = 48 \text{ cm}^2$$

$$\therefore \quad \text{Actual area} = k^2 \times \text{Area of ABCD}$$

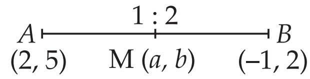
$$= (50{,}000)^2 \times 48 \text{ cm}^2$$

$$= \frac{50{,}000 \times 50{,}000 \times 48}{1{,}00{,}000 \times 1{,}00{,}000} \text{ km}^2$$

$$= 12 \text{ km}^2.$$ **Ans.**

(c) Given, vertices of triangle are, A (2, 5), B (– 1, 2), C (5, 8), AM : MB = 1 : 2.

$\because$ M is a point on AB.



$$\therefore \text{Coordinates of M} = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

Here, $m_1 : m_2 = 1 : 2, x_1 = 2, y_1 = 5, x_2 = -1, y_2 = 2$

$\therefore$ Coordinates of M

$$= \left(\frac{1 \times (-1) + 2 \times 2}{1 + 2}, \frac{1 \times 2 + 2 \times 5}{1 + 2}\right)$$

$$= \left(\frac{-1 + 4}{3}, \frac{12}{3}\right) = (1, 4)$$ **Ans.**

The equation of line passing through C (5, 8) and M (1, 4) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Here, $x_1 = 5, y_1 = 8, x_2 = 1, y_2 = 4$

$$\therefore \quad y - 8 = \frac{4 - 8}{1 - 5}(x - 5)$$

$$\Rightarrow \quad y - 8 = \frac{-4}{-4}(x - 5)$$

$$\Rightarrow \quad y - 8 = x - 5$$

$$\Rightarrow \quad x - y + 3 = 0.$$ **Ans.**

Solution 8.

(a) Let the original number of children be x.

Total amount to be distributed = ₹ 7,500

$$\therefore \quad \text{Each will receive} = \frac{7{,}500}{x}$$

If the number of children are $(x - 20)$

$$\text{Then, each will receive} = \frac{7{,}500}{x - 20}$$

According to question,

$$\frac{7{,}500}{x - 20} - \frac{7{,}500}{x} = 100$$

$$\Rightarrow \quad 7{,}500\left(\frac{1}{x - 20} - \frac{1}{x}\right) = 100$$

$$\Rightarrow \quad \frac{x - x + 20}{x(x - 20)} = \frac{100}{7{,}500}$$

$$\Rightarrow \quad \frac{20}{x^2 - 20x} = \frac{1}{75}$$

$$\Rightarrow \quad x^2 - 20x = 1{,}500$$

$$\Rightarrow \quad x^2 - 20x - 1{,}500 = 0$$

$$\Rightarrow \quad x^2 - (50 - 30)x - 1{,}500 = 0$$

$$\Rightarrow \quad x^2 - 50x + 30x - 1{,}500 = 0$$

$$\Rightarrow \quad x(x - 50) + 30(x - 50) = 0$$

$$\Rightarrow \quad (x - 50)(x + 30) = 0$$

$$\Rightarrow \quad x - 50 = 0 \text{ or } x + 30 = 0$$

$$\Rightarrow \quad x = 50 \text{ or } x = -30$$

$$\therefore \quad x = 50$$

($\because x$ cannot be negative)

$\therefore$ The original number of children = 50. **Ans.**

(b)

Marks	Mid values (x)	No. of students (f)	fx
0 – 10	5	7	35
10 – 20	15	a	15a
20 – 30	25	8	200
30 – 40	35	10	350
40 – 50	45	5	225
		$\sum f = 30 + a$	$\sum fx = 15a + 810$

$\therefore$ Mean $= \frac{\Sigma fx}{\Sigma f}$

$\Rightarrow$ $24 = \frac{15a + 810}{a + 30}$

$\Rightarrow$ $24a + 720 = 15a + 810$

$\Rightarrow$ $24a - 15a = 810 - 720$

$\Rightarrow$ $9a = 90$

$\Rightarrow$ $a = 10.$ **Ans.**

(c) Given, BC = 5 cm, AB = 6.5 cm, $\angle$ABC = 120°

Steps of construction :

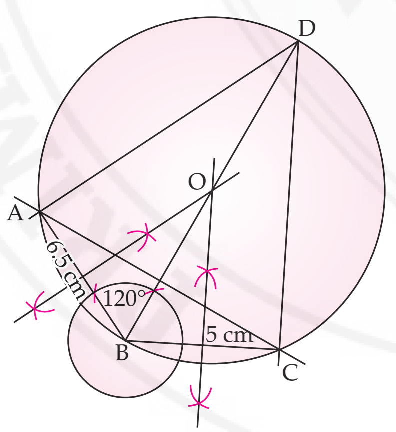
(i) Construct ΔABC with given data.

(ii) Draw perpendicular bisectors of BC and AB which meet at O.

(iii) Taking O as centre and OB as radius, draw circumcircle of ΔABC passing through A, B and C.

(iv) Draw angle bisector of ABC as BD which meets circle at D.

(v) Join AD and CD. ABCD is the required cyclic quadrilateral.



Solution 9.

(a) Given, P = ₹ 1,000, r = 10%, I = ₹ 5,550, n = ?

$\therefore$ $I = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$

$\Rightarrow$ $5550 = 1000 \times \frac{n^2 + n}{24} \times \frac{10}{100}$

$\Rightarrow$ $555 = \frac{5}{12}(n^2 + n)$

$\Rightarrow$ $5n^2 + 5n = 6660$

$\Rightarrow$ $5(n^2 + n) = 6660$

$\Rightarrow$ $n^2 + n = 1332$

$\Rightarrow$ $n^2 + n - 1332 = 0$

$\Rightarrow$ $n^2 + 37n - 36n - 1332 = 0$

$\Rightarrow$ $n(n + 37) - 36(n + 37) = 0$

$\Rightarrow$ $(n + 37)(n - 36) = 0$

$\Rightarrow$ $n + 37 = 0$ or $n - 36 = 0$

$\Rightarrow$ $n = -37$ or $n = 36$

$\therefore$ $n = 36$

($\because$ n cannot be negative)

Hence, total time for which amount was held is 36 months or 3 years. **Ans.**

(b) Given, MN || QR, and $\frac{PM}{MQ} = \frac{2}{3}$.

Now, $\frac{PM}{MQ} = \frac{2}{3}$

$\Rightarrow$ $\frac{PM}{PM + MQ} = \frac{2}{2+3}$

$\Rightarrow$ $\frac{PM}{PQ} = \frac{2}{5}$

(i) In ΔPMN and ΔPQR,

$\angle P = \angle P$ (common angle)

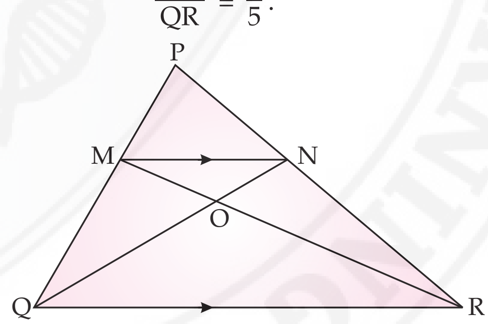
$\angle PMN = \angle PQR$

(corresponding angles, MN || QR)

$\therefore$ $\Delta PMN \sim \Delta PQR$ (AA axiom)

$\therefore$ $\frac{MN}{QR} = \frac{PM}{PQ}$ (corresponding sides of similar Δs)

$\frac{MN}{QR} = \frac{2}{5}$. **Ans.**



(ii) In ΔOMN and ΔORQ,

$\angle MON = \angle QOR$

(vertically opposite angles)

$\angle OMN = \angle ORQ$

(Alternate angles, MN || QR)

$\therefore$ $\Delta OMN \sim \Delta ORQ$ (By AA axiom)

Ans.

(iii) $\frac{\text{Area of } (\Delta OMN)}{\text{Area of } (\Delta ORQ)} = \frac{MN^2}{QR^2}$

(Area of similar triangles are proportional to the square of their corresponding sides)

$$= \frac{2^2}{5^2} = \frac{4}{25}$$

$\Rightarrow$ Area of ΔOMN : Area of $\Delta ORQ = 4 : 25$. **Ans.**

(c) Given, common radius $(r) = 7$ cm.

Height of cylinder = Height of cone

$= h = 4$ cm.

$\therefore$ Volume of solid = Volume of cone + Volume of cylinder + Volume of hemisphere

$$= \frac{1}{3}\pi r^2 h + \pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \pi r^2\left(\frac{1}{3}h + h + \frac{2}{3}r\right)$$

$$= \frac{22}{7} \times 7^2\left(\frac{1}{3} \times 4 + 4 + \frac{2}{3} \times 7\right)$$

$$= 22 \times 7\left(\frac{4}{3} + 4 + \frac{14}{3}\right)$$

$$= 154\left(\frac{4 + 12 + 14}{3}\right)$$

$$= 154\left(\frac{30}{3}\right)$$

$$= 154 \times 10 = 1540 \text{ cm}^3.$$ **Ans.**

Solution 10.

(a) Let $f(x) = 2x^3 + 3x^2 - 9x - 10$

For $x = 2$,

$$f(2) = 2 \times 2^3 + 3 \times 2^2 - 9 \times 2 - 10$$
$$= 16 + 12 - 18 - 10$$
$$= 28 - 28 = 0.$$

$\therefore$ $(x - 2)$ is a factor of $f(x)$

$$x - 2)\ 2x^3 + 3x^2 - 9x - 10\ (2x^2 + 7x + 5$$
$$2x^3 - 4x^2$$
$$-\quad +$$
$$7x^2 - 9x$$
$$7x^2 - 14x$$
$$-\quad +$$
$$5x - 10$$
$$5x - 10$$
$$-\quad +$$
$$\times$$

Now, $2x^2 + 7x + 5 = 2x^2 + 5x + 2x + 5$

$= x(2x + 5) + 1(2x + 5)$

$= (2x + 5)(x + 1)$

$\therefore$ $f(x) = (x + 1)(x - 2)(2x + 5)$ **Ans.**

(b) Given, QR = OP, $\angle$ORP = 20°

But, OP = OQ (radius of circle)

$\Rightarrow$ OP = OQ = QR

$\therefore$ $\angle$QOS = $\angle$ORQ ($\because$ QR = OQ)

= 20°.

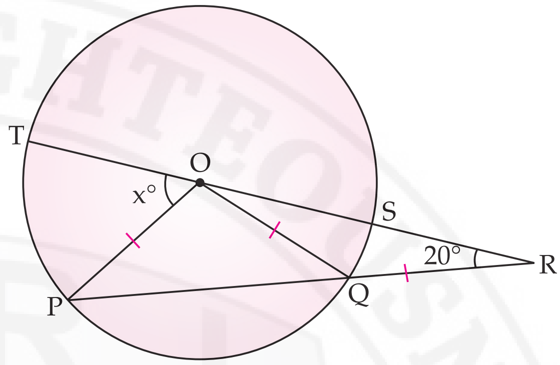
$\therefore$ $\angle$OQP = $\angle$QOR + $\angle$ORQ

(Exterior angle is equal to sum of interior opposite angles)

= 20° + 20° = 40°

$\therefore$ $\angle$OPQ = $\angle$OQP ($\because$ OP = OQ)

= 40°



$\therefore$ $\angle$POQ + $\angle$OPQ + $\angle$OQP = 180°

(sum of angles in a triangle is180°)

$\Rightarrow$ $\angle$POQ + 40° + 40° = 180°

$\Rightarrow$ $\angle$POQ = 180° – 80° = 100°

$\therefore$ $\angle$POT + $\angle$POQ + $\angle$QOR = 180°

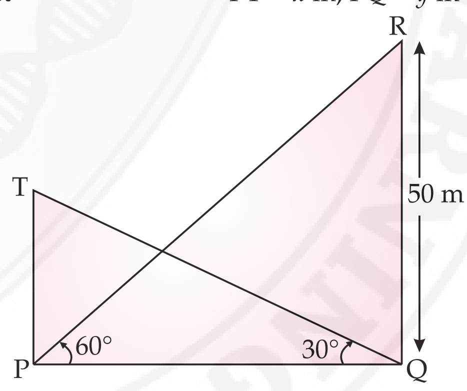
(sum of angles on a straight line is 180°)

$\Rightarrow$ $x° + 100° + 20° = 180°$

$\Rightarrow$ $x° = 180° - 120° = 60°$. **Ans.**

(c) Given, QR = 50 m, $\angle$RPQ = 60°, $\angle$PQT = 30°.

Let PT = x m, PQ = y m



$\therefore$ In ΔPQR, $\tan 60° = \frac{QR}{PQ}$

$\Rightarrow$ $\sqrt{3} = \frac{50}{y}$

$\Rightarrow$ $y = \frac{50}{\sqrt{3}}$ ...(i)

In ΔPQT, $\tan 30° = \frac{PT}{PQ}$

$\Rightarrow$ $\frac{1}{\sqrt{3}} = \frac{x}{y}$

$\Rightarrow \quad x = \frac{y}{\sqrt{3}}$ ...(ii)

$= \frac{\frac{50}{\sqrt{3}}}{\sqrt{3}}$ [using eqn. (i)]

$= \frac{50}{\sqrt{3} \times \sqrt{3}} = \frac{50}{3}$

$= 16.667$

$= 17$ m **Ans.**

(correct to the nearest metre)

Solution 11.

(a) Let a be the first term and d be the common difference of given A.P.

$\therefore \quad a_4 = 22$ and $a_{15} = 66$ (Given)

$\Rightarrow \quad a + 3d = 22$ ...(i)

and $\quad a + 14d = 66$ ...(ii)

Subtracting equation (i) from equation (ii), we get

$$\begin{array}{r} a + 14d = 66 \\ a + 3d = 22 \\ - \quad - \quad - \\ \hline 11d = 44 \end{array}$$

$\Rightarrow \quad d = 4$

From equation (i),

$a + 3 \times 4 = 22$

$\Rightarrow \quad a = 22 - 12 = 10$

$\therefore \quad a = 10, d = 4.$

Sum of series to 8 terms,

$S_8 = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{8}{2}[2 \times 10 + (8 - 1)4]$

$= 4(20 + 28)$

$= 4 \times 48 = 192$ **Ans.**

(b)

Height in cm	No. of Boys	*c.f.*
135 – 140	4	4
140 – 145	8	12
145 – 150	20	32
150 – 155	14	46
155 – 160	7	53
160 – 165	6	59
165 – 170	1	60
	$n = 60$	

(i) Median $= \frac{n}{2}$th observation

$= \frac{60}{2}$th observation

$= 30$th observation

$= 150$ cm (from ogive) **Ans.**

(ii) Lower quartile $= \frac{n}{4}$th observation

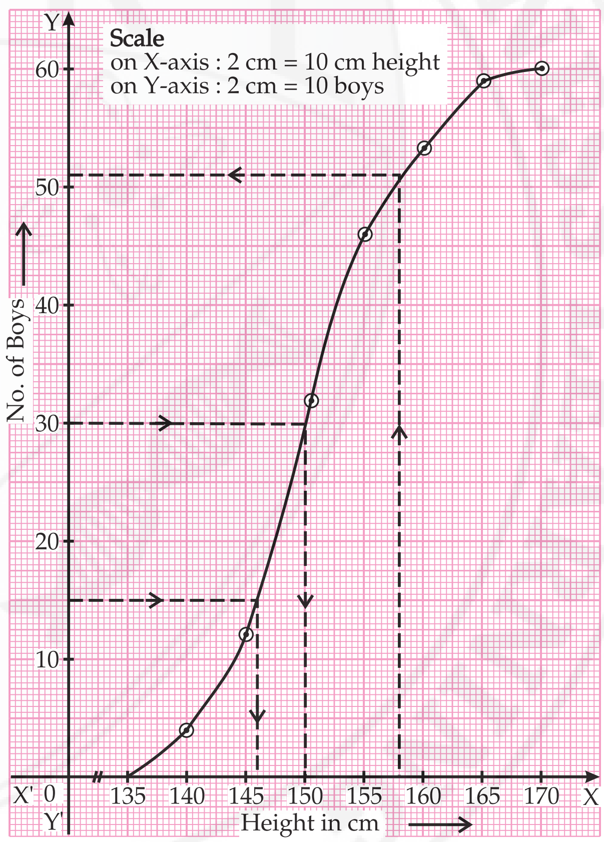
$= \frac{60}{4}$th observation

$= 15$th observation

$= 146$ cm (from ogive) **Ans.**

(iii) No. of boys whose height is less than 158 cm = 51. (from ogive)

$\therefore$ No. of tall boys $= 60 - 51 = 9.$ **Ans.**



MATHEMATICS 2017

QUESTIONS

SECTION—A (40 Marks)

*(Attempt **all** questions from this Section)*

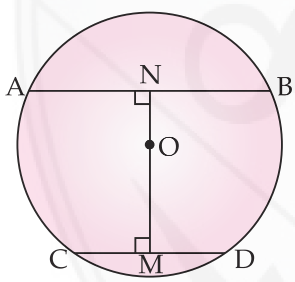
Question 1.

(a) *If b is the mean proportion between a and c, show that:* **[3]**

$$\frac{a^4+a^2b^2+b^4}{b^4+b^2c^2+c^4} = \frac{a^2}{c^2}$$

(b) *Solve the equation $4x^2 - 5x - 3 = 0$ and give your answer correct to two decimal places.* **[4]**

(c) *AB and CD are two parallel chords of a circle such that AB = 24 cm and CD = 10 cm. If the radius of the circle is 13 cm, find the distance between the two chords.*** **[3]**



Question 2.

(a) *Evaluate without using trigonometric tables,*** **[3]**

$$\sin^2 28° + \sin^2 62° + \tan^2 38° - \cot^2 52° + \frac{1}{4}\sec^2 30°$$

(b) *If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$ and $A^2 - 5B^2 = 5C$. Find matrix C where C is a 2 by 2 matrix.* **[4]**

(c) *Jaya borrowed ₹ 50,000 for 2 years. The rates of interest for two successive years are 12% and 15% respectively. She repays ₹ 33,000 at the end of the first year. Find the amount she must pay at the end of the second year to clear her debt.*** **[3]**

Question 3.

(a) *The catalogue price of a computer set is ₹ 42000. The shopkeeper gives a discount of 10% on the listed price. He further gives an off-season discount of 5% on the discounted price. However, sales tax at 8% is charged on the remaining price after the two successive discounts. Find :*** **[3]**

(i) *the amount of sales tax a customer has to pay*

(ii) *the total price to be paid by the customer for the computer set.*

(b) *P(1, – 2) is a point on the line segment AB with A(3, – 6) and B(x, y) such that AP : PB is equal to 2 : 3. Find the coordinates of B.* **[4]**

(c) *The marks of 10 students of a class in an examination arranged in ascending order are as follows :* **[3]**

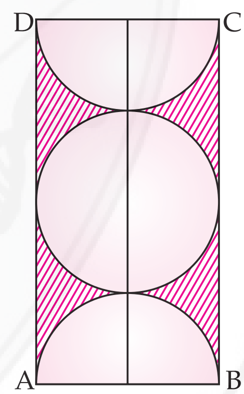
13, 35, 43, 46, x, x + 4, 55, 61, 71, 80

If the median marks is 48, find the value of x. Hence find the mode of the given data.

Question 4.

(a) *What must be subtracted from $16x^3 - 8x^2 + 4x + 7$ so that the resulting expression has $2x + 1$ as a factor ?* **[3]**

(b) *In the given figure ABCD is a rectangle. It consists of a circle and two semi-circles each of which are of radius 5 cm. Find the area of the shaded region. Give your answer correct to three significant figures.*** **[4]**



(c) *Solve the following inequation and represent the solution set on a number line.* **[3]**

$$-8\frac{1}{2} < -\frac{1}{2} - 4x \le 7\frac{1}{2}, x \in I$$

SECTION—B (40 Marks)

*(Attempt any **four** questions from this Section)*

Question 5.

(a) *Given matrix $B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$. Find the matrix X if, $X = B^2 - 4B$.* **[4]**

Hence, solve for a and b, given $X\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

(b) *How much should a man invest in ₹ 50 shares selling at ₹ 60 to obtain an income of ₹450, if the rate of dividend declared is 10%. Also find his yield percent, to the nearest whole number.* **[3]**

** Answer is not given due to change in the present syllabus.

(c) *Sixteen cards are labelled as a, b, c, ……… m, n, o, p. They are put in a box and shuffled. A boy is asked to draw a card from the box. What is the probability that the card drawn is :* **[3]**

(i) *a vowel.*

(ii) *a consonant.*

(iii) *none of the letters of the word 'median'.*

Question 6.

(a) *Using a ruler and a compass, construct a triangle ABC in which AB = 7cm, ∠ CAB = 60° and AC = 5 cm. Construct the locus of :* **[4]**

(i) *points equidistant from AB and AC.*

(ii) *points equidistant from BA and BC.*

Hence, construct a circle touching the three sides of the triangle internally.

(b) *A conical tent has to accommodate 77 persons. Each person must have 16 m^3 of air to breathe. Given the radius of the tent as 7m, find the height of the tent and also its curved surface area.* **[3]**

(c) *If* $\frac{7m+2n}{7m-2n}=\frac{5}{3}$ *use properties of proportion to find* **[3]**

(i) $m : n$

(ii) $\frac{m^2+n^2}{m^2-n^2}$

Question 7.

(a) *A page from a savings bank account passbook is given below :*** **[5]**

Data	Particulars	Amount Withdrawn (₹)	Amount Deposited (₹)	Balance (₹)
Jan. 7, 2016	B/F			3000.00
Jan. 10, 2016	By Cheque		2600.00	5600.00
Feb. 8, 2016	To Self	1500.00		4100.00
Apr. 6, 2016	By Cheque	2100.00		2000.00
May 4, 2016	By Cash		6500.00	8500.00
May 27, 2016	By Cheque		1500.00	10000.00

(i) *Calculate the interest for the 6 months from January to June 2016, at 6% per annum.*

(ii) *If the account is closed on 1st July 2016, find the amount received by the account holder.*

(b) *Use a graph paper for this question (Take 2 cm = 1 unit on both X and Y axis)* **[5]**

(i) *Plot the following points :*

A(0, 4), B(2, 3), C(1, 1) and D(2, 0)

(ii) *Reflect points B, C, D on the Y-axis and write down their coordinates. Name the images as B′, C′, D′ respectively.*

(iii) *Join the points A, B, C, D, D′, C′, B′ and A in order, so as form a closed figure.*

*Write down equation of the line of symmetry of the figure formed.***

Question 8.

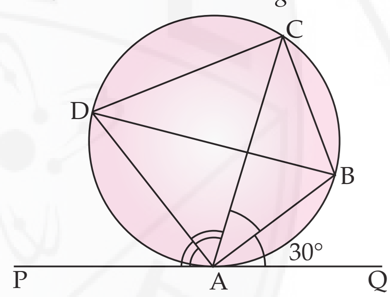
(a) *Calculate the mean of the following distribution using step deviation method.* **[4]**

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of Students	10	9	25	30	16	10

(b) *In the given figure PQ is a tangent to the circle at A. AB and AD are bisectors of ∠CAQ and ∠PAC. If ∠BAQ = 30°, prove that :* **[3]**

(i) *BD is a diameter of the circle.*

(ii) *ΔABC is an isosceles triangle.*



(c) *The printed price of an air conditioner is ₹ 45,000.The wholesaler allows a discount of 10% to the shopkeeper. The shopkeeper sells the article to the customer at a discount of 5% of the marked price. Sales tax (under VAT) is charged at the rate of 12% at every stage. Find :*** **[3]**

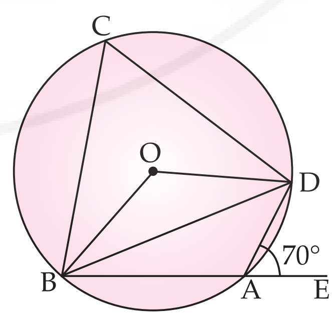
(i) *VAT paid by the shopkeeper to the government.*

(ii) *The total amount paid by the customer inclusive of tax.*

Question 9.

(a) *In the figure given, O is the centre of the circle. ∠DAE= 70°. Find, giving suitable reasons, the measure of :* **[4]**

(i) *∠BCD* **(ii)** *∠BOD* **(iii)** *∠OBD*



(b) *A(– 1, 3), B(4, 2) and C(3, – 2) are the vertices of a triangle.* **[3]**

** Answer is not given due to change in the present syllabus.

(i) *Find the coordinates of the centroid G of the triangle.*

(ii) *Find the equation of the line through G and parallel to AC.*

(c) *Prove that* **[3]**

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$$

Question 10.

(a) *The sum of the ages of Vivek and his younger brother Amit is 47 years. The product of their ages in years is 550. Find their ages.* **[4]**

(b) *The daily wages of 80 workers in a project are given below.* **[6]**

Wages (in ₹)	400–450	450–500	500–550	550–600	600–650	650–700	700–750
No. of workers	2	6	12	18	24	13	5

Use a graph paper to draw an ogive for the above distribution. (Use a scale of 2 cm = ₹ 50 on X-axis and 2 cm = 10 workers on Y-axis). Use your ogive to estimate :

(i) *the median wage of the workers.*

(ii) *the lower quartile wage of workers.*

(iii) *the number of workers who earn more than ₹ 625 daily.* **[6]**

Question 11.

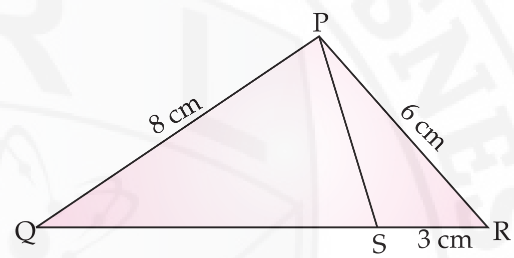
(a) *The angles of depression of two ships A and B as observed from the top of a light house 60 m high are 60° and 45° respectively. If the two ships are on the opposite sides of the light house, find the distance between the two ships. Give your answer correct to the nearest whole number.* **[4]**

(b) *PQR is a triangle. S is a point on the side QR of ΔPQR such that ∠PSR = ∠QPR. Given QP = 8 cm, PR = 6 cm and SR = 3 cm* **[3]**

(i) *Prove ΔPQR ~ ΔSPR*

(ii) *Find the length of QR and PS*

(iii) $\dfrac{\text{Area of } \Delta PQR}{\text{Area of } \Delta SPR}$



(c) *Mr. Richard has a recurring deposit account in a bank for 3 years at 7.5% p.a. simple interest. If he gets ₹ 8325 as interest at the time of maturity, find :* **[3]**

(i) *The monthly deposit*

(ii) *The maturity value.*

Answers

SECTION—A

Solution 1.

(a) Given, b is mean proportion between a and c.

$\therefore \quad \frac{a}{b} = \frac{b}{c} = k$ (say)

$\Rightarrow \quad b = kc;\ a = kb = k(kc) = k^2c$

$$\therefore \quad \text{L.H.S.} = \frac{a^4 + a^2b^2 + b^4}{b^4 + b^2c^2 + c^4}$$

$$= \frac{(k^2c)^4 + (k^2c)^2.(kc)^2 + (kc)^4}{(kc)^4 + (kc)^2c^2 + c^4}$$

$$= \frac{k^8c^4 + k^6c^4 + k^4c^4}{k^4c^4 + k^2c^4 + c^4}$$

$$= \frac{k^4c^4(k^4 + k^2 + 1)}{c^4(k^4 + k^2 + 1)} = k^4$$

and $\quad \text{R.H.S.} = \frac{a^2}{c^2} = \frac{(k^2c)^2}{c^2} = \frac{k^4c^2}{c^2} = k^4$

$\therefore \quad$ L.H.S. = R.H.S. **Hence Proved.**

(b) Given equation is, $4x^2 - 5x - 3 = 0$.

Comparing it with $ax^2 + bx + c = 0$, we have

$a = 4, b = -5, c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 4 \times (-3)}}{2 \times 4}$$

$$= \frac{5 \pm \sqrt{25 + 48}}{8} = \frac{5 \pm \sqrt{73}}{8}$$

$$= \frac{5 \pm 8.544}{8}$$

$$= \frac{5 + 8.544}{8} \text{ or } \frac{5 - 8.544}{8}$$

$$= \frac{13.544}{8} \text{ or } \frac{-3.544}{8}$$

$= 1{\cdot}693$ or $-0{\cdot}443$

$= 1{\cdot}69$ or $-0{\cdot}44$

(correct to 2 decimal places) **Ans.**

Solution 2.

(b) Given, $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$

We have, $\quad A^2 - 5B^2 = 5C$

$$\Rightarrow \begin{bmatrix}1 & 3\\3 & 4\end{bmatrix}\begin{bmatrix}1 & 3\\3 & 4\end{bmatrix} - 5\begin{bmatrix}-2 & 1\\-3 & 2\end{bmatrix}\begin{bmatrix}-2 & 1\\-3 & 2\end{bmatrix} = 5C$$

$$\Rightarrow 5C = \begin{bmatrix}1+9 & 3+12\\3+12 & 9+16\end{bmatrix} - 5\begin{bmatrix}4-3 & -2+2\\6-6 & -3+4\end{bmatrix}$$

$$\Rightarrow 5C = \begin{bmatrix}10 & 15\\15 & 25\end{bmatrix} - 5\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$\Rightarrow 5C = \begin{bmatrix}10 & 15\\15 & 25\end{bmatrix} - \begin{bmatrix}5 & 0\\0 & 5\end{bmatrix}$$

$$\Rightarrow 5C = \begin{bmatrix}5 & 15\\15 & 20\end{bmatrix}$$

$$\Rightarrow C = \frac{1}{5}\begin{bmatrix}5 & 15\\15 & 20\end{bmatrix}$$

$$\Rightarrow C = \begin{bmatrix}1 & 3\\3 & 4\end{bmatrix}$$ **Ans.**

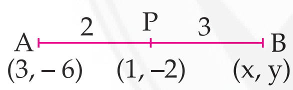
Solution 3.

(b) Given Co-ordinates are P (1, – 2), A (3, – 6), B (x, y), and AP : PB = 2 : 3

By section formula,

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2},$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2},$$



$$\Rightarrow 1 = \frac{2 \times x + 3 \times 3}{2+3}, \quad -2 = \frac{2 \times y + 3 \times (-6)}{2+3}$$

$$\Rightarrow 5 = 2x + 9, \quad -10 = 2y - 18$$

$$\Rightarrow 2x = 5 - 9, \quad 2y = -10 + 18$$

$$\Rightarrow x = \frac{-4}{2}, \quad y = \frac{8}{2}$$

$$\Rightarrow x = -2, y = 4$$

The coordinates of B are (– 2, 4) **Ans.**

(c) Given marks are 13, 35, 43, 46, x, $x + 4$, 55, 61, 71, 80.

Median = 48

$\because n = 10$ (even)

$$\therefore \text{Median} = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th}\text{observation} + \left(\frac{n}{2}+1\right)^{th}\text{observation}\right]$$

$$= \frac{5^{th}\text{observation} + 6^{th}\text{observation}}{2}$$

$$= \frac{x + x + 4}{2}$$

$$\therefore 48 = \frac{2x+4}{2}$$

$$\Rightarrow 48 = \frac{2(x+2)}{2}$$

$$\Rightarrow x + 2 = 48$$

$$\Rightarrow x = 48 - 2 = 46$$

$$\therefore x + 4 = 46 + 4 = 50$$

∴ The marks are : 13, 35, 43, 46, 46, 50, 55, 61, 71, 80.

Since 46 has highest frequency

∴ Mode = 46 **Ans.**

Solution 4.

(a) Let the required number be K.

Let $f(x) = 16x^3 - 8x^2 + 4x + 7 - K$

$\because$ $(2x + 1)$ is a factor of $f(x)$

$$\therefore f\left(-\frac{1}{2}\right) = 0$$

$$\Rightarrow 16 \times \left(-\frac{1}{2}\right)^3 - 8 \times \left(-\frac{1}{2}\right)^2 + 4 \times \left(-\frac{1}{2}\right) + 7 - K = 0$$

$$\Rightarrow -16 \times \frac{1}{8} - 8 \times \frac{1}{4} - 4 \times \frac{1}{2} + 7 - K = 0$$

$$\Rightarrow -2 - 2 - 2 + 7 - K = 0$$

$$\Rightarrow -6 + 7 - K = 0$$

$$\Rightarrow 1 - K = 0$$

$$\Rightarrow K = 1$$

∴ The required number to be subtracted is 1. **Ans.**

(c) Given inequation is,

$$-8\frac{1}{2} < -\frac{1}{2} - 4x \le 7\frac{1}{2}, x \in I$$

$$\Rightarrow -\frac{17}{2} < \frac{-1-8x}{2} \le \frac{15}{2}$$

$$\Rightarrow -\frac{17}{2} \times 2 < \frac{-1-8x}{2} \times 2 \le \frac{15}{2} \times 2$$

[Multiplying with 2 in complete inequation]

$$\Rightarrow -17 < -1 - 8x \le 15$$

$$\Rightarrow -17 + 1 < -1 + 1 - 8x \le 15 + 1$$

[Adding 1 in complete inequation]

$$\Rightarrow -16 < -8x \le 16$$

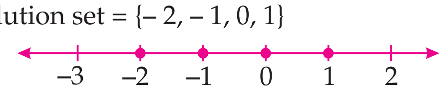
$$\Rightarrow \frac{-16}{-8} > \frac{-8x}{-8} \ge \frac{16}{-8}$$

[Dividing by – 8 in the inequation]

$$\Rightarrow 2 > x \ge -2$$

$$\Rightarrow -2 \le x < 2$$

∴ Solution set = {– 2, – 1, 0, 1}



Ans.

SECTION—B

Solution 5.

(a) Given, $B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$

$\therefore \quad X = B^2 - 4B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1+8 & 1+3 \\ 8+24 & 8+9 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$

$= \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$

$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ **Ans.**

Now, $X\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 5a+0 \\ 0+5b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 5a \\ 5b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

On comparing the elements of matrices on both the sides, we get

$5a = 5$ and $5b = 50$

$\Rightarrow \quad a = 1$ and $b = 10$ **Ans.**

(b) Given, N.V. = ₹ 50, M.V. = ₹ 60,

Total income = ₹ 450, Rate of dividend = 10%

$\therefore$ Dividend per share

$= \frac{10}{100} \times$ ₹ 50 = ₹ 5

$\therefore$ Number of shares, (n)

$= \frac{₹\,450}{₹\,5} = 90$

$\therefore$ Investment = M.V. $\times\ n$

= ₹ 60 × 90 = ₹ 5400 **Ans.**

$\therefore$ Yield percent $= \frac{₹\,5}{₹\,60} \times 100\%$

$= \frac{25}{3} = 8{\cdot}33\%$

= 8% (to the nearest whole no.)

Ans.

(c) Here, sample space,

$(S) = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p\}$

$\therefore \quad n(S) = 16$

(i) Vowels, $V = \{a, e, i, o\}$

$\therefore \quad n(V) = 4$

$\therefore \quad P(a \text{ vowel}) = \frac{n(V)}{n(S)}$

$= \frac{4}{16} = \frac{1}{4}$ **Ans.**

(ii) Consonants,

$C = \{b, c, d, f, g, h, j, k, l, m, n, p\}$

$\therefore \quad n(C) = 12$

$\therefore \quad P(a \text{ consonant}) = \frac{n(C)}{n(S)}$

$= \frac{12}{16} = \frac{3}{4}$ **Ans.**

(iii) None of the letters of the word 'median'

$(N) = \{b, c, f, g, h, j, k, l, o, p\}$

$\therefore \quad n(N) = 10$

$\therefore \quad P(N) = \frac{n(N)}{n(S)}$

$= \frac{10}{16} = \frac{5}{8}$ **Ans.**

Solution 6.

(a) Given, AB = 7 cm, $\angle$ CAB = 60°, AC = 5 cm

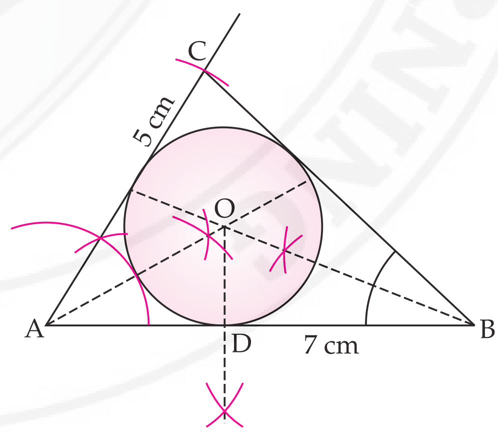
Steps of construction :

1. Construct triangle ABC with given measurements.

2. Draw bisector of $\angle$ BAC which is the locus of points equidistant from AB and AC.

3. Draw bisector of $\angle$ ABC which is the locus of points equidistant from BA and BC.

4. Let the bisectors meet at O.



5. Draw a perpendicular from O to AB intersecting AB at D.

6. Taking O as centre and OD as radius, construct a circle touching the three sides of triangle internally.

(i) Bisector of $\angle$ A **(ii)** Bisector of $\angle$ B

(b) Given, number of persons = 77.

Volume of air required by each person = 16 m^3

$\therefore$ Total volume of air required for 77 persons $= 77 \times 16\ m^3 = 1232\ m^3$.

Radius (r) = 7 m

Let the height of tent be h m.

Then, Volume of tent $= \frac{1}{3}\pi r^2 h$

$$\Rightarrow \quad 1232 = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times h$$

$$\Rightarrow \quad h = \frac{1232 \times 3}{22 \times 7} = 24\ m$$

$\therefore$ Required height = 24 m

Now, slant height (l) $= \sqrt{h^2 + r^2}$

$$= \sqrt{24^2 + 7^2} = \sqrt{576 + 49}$$

$$= \sqrt{625} = 25\ m$$

$\therefore$ The curved surface area

$$= \pi r l$$

$$= \frac{22}{7} \times 7 \times 25$$

$$= 550\ m^2$$ **Ans.**

(c) **(i)** Given, $\frac{7m + 2n}{7m - 2n} = \frac{5}{3}$

Using componendo and dividendo,

$$\frac{(7m + 2n) + (7m - 2n)}{(7m + 2n) - (7m - 2n)} = \frac{5 + 3}{5 - 3}$$

$$\Rightarrow \quad \frac{7m + 7m}{2n + 2n} = \frac{8}{2}$$

$$\Rightarrow \quad \frac{14m}{4n} = \frac{4}{1}$$

$$\Rightarrow \quad \frac{7m}{2n} = \frac{4}{1}$$

$$\Rightarrow \quad \frac{m}{n} = \frac{4}{1} \times \frac{2}{7}$$

$$\Rightarrow \quad m : n = 8 : 7$$ **Ans.**

(ii) $\frac{m}{n} = \frac{8}{7} \Rightarrow \frac{m^2}{n^2} = \frac{64}{49}$

Using componendo and dividendo,

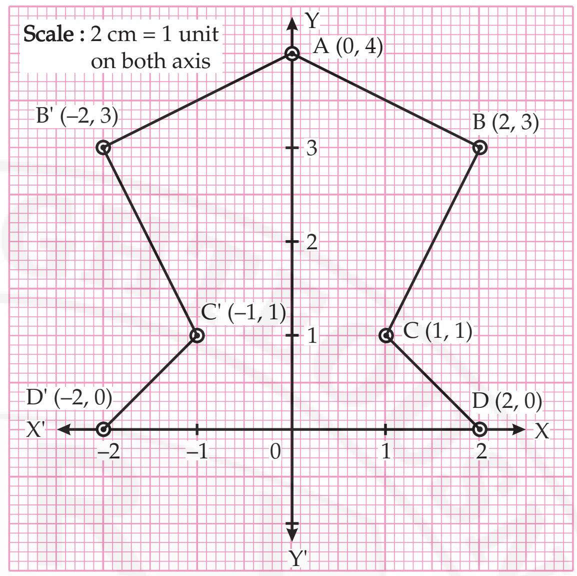
$$\frac{m^2 + n^2}{m^2 - n^2} = \frac{64 + 49}{64 - 49} = \frac{113}{15}$$ **Ans.**

Solution 7.

(b) **(i)** On graph : A (0, 4), B (2, 3), C (1, 1), D (2, 0)

(ii) B′ (– 2, 3), C′ (– 1, 1), D′ (– 2, 0)

(iii)



Solution 8.

(a)

Marks	Mid values (x_i)	No. of students (f_i)	$d_i = x_i$ – A	$t_i = \frac{d_i}{h}$	$f_i t_i$
0–10	5	10	– 20	– 2	– 20
10–20	15	9	– 10	– 1	– 9
20–30	25 = A	25	0	0	0
30–40	35	30	10	1	30
40–50	45	16	20	2	32
50–60	55	10	30	3	30
		Σf_i = 100			$\Sigma f_i t_i$ = 63

Let, A = 25 and h = 10

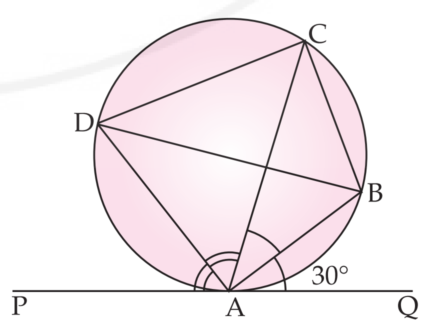
$$\therefore \quad \text{Mean} = A + \frac{\Sigma f_i t_i}{\Sigma f_i} \times h$$

$$= 25 + \frac{63}{100} \times 10$$

$$= 25 + 6{\cdot}3$$

$$= 31{\cdot}3$$ **Ans.**

(b) Given, $\angle$BAQ = 30°, AB and AD are bisectors of $\angle$CAQ and $\angle$PAC.



(i) $\angle BAC = \angle BAQ = 30°$

(AB bisects $\angle CAQ$)

$\angle CAQ = \angle BAC + \angle BAQ$

$= 30° + 30° = 60°$

$\angle PAC = 180° - \angle CAQ$

(Linear pair)

$= 180° - 60° = 120°$

$\angle CAD = \frac{1}{2} \angle PAC$

(AD bisects $\angle PAC$)

$= \frac{1}{2} \times 120° = 60°$

$\angle BAD = \angle BAC + \angle CAD$

$= 30° + 60° = 90°$

$\therefore$ BD is a diameter ($\because \angle BAD = 90°$ = angle in a semi-circle) **Hence Proved.**

(ii) $\angle ADB = \angle BAC = 30°$

(angles in an alternate segment are equal)

$\angle ACB = \angle ADB$

(angles in same segment are equal)

$\therefore$ $\angle BAC = \angle ACB = 30°$

$\therefore$ AB = BC

(sides opposite to equal angles are equal)

$\therefore$ ΔABC is an isosceles triangle. **Hence Proved.**

Solution 9.

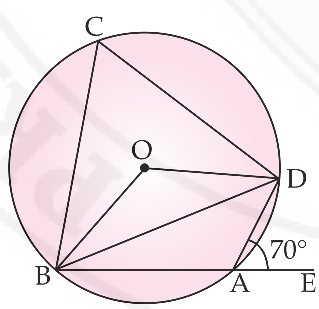
(a) Given, $\angle DAE = 70°$

(i) $\angle BAD + \angle DAE = 180°$ (Linear pair)

$\Rightarrow$ $\angle BAD = 180° - 70° = 110°$

Now, $\angle BCD + \angle BAD = 180°$.

(Sum of opposite angles of cyclic quadrilateral is 180°)



$\Rightarrow$ $\angle BCD = 180° - 110°$

$= 70°$ **Ans.**

(ii) $\angle BOD = 2 \angle BCD$

(Angle that an arc subtends at the centre is twice the angle at circumference of the circle)

$= 2 \times 70°$

$= 140°$ **Ans.**

(iii) $\angle OBD = \angle ODB$

(OB = OD = radius)

$\therefore$ $\angle OBD + \angle ODB + \angle BOD = 180°$

(Sum of angles in a triangle is 180°)

$\Rightarrow$ $\angle OBD + \angle OBD + 140° = 180°$

($\because \angle OBD = \angle ODB$)

$\Rightarrow$ $2\angle OBD = 180° - 140°$

$\Rightarrow$ $\angle OBD = \frac{40°}{2} = 20°$ **Ans.**

(b) Given, A (– 1, 3), B (4, 2), C (3, – 2).

(i) Coordinates of centroid

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$= \left(\frac{-1+4+3}{3}, \frac{3+2-2}{3}\right)$$

$$= \left(\frac{6}{3}, \frac{3}{3}\right) = (2, 1)$$ **Ans.**

(ii) Slope of AC $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-3}{3-(-1)} = \frac{-5}{4}$

Since, required line and the segment joining points A and C are parallel, so their slopes will be equal.

$\therefore$ Slope of the required line $(m) = \frac{-5}{4}$

Let the equation of the line through G, be

$$y - y_1 = m(x - x_1)$$

$\Rightarrow$ $y - 1 = -\frac{5}{4}(x - 2)$

$\Rightarrow$ $4y - 4 = -5x + 10$

$\Rightarrow$ $5x + 4y - 14 = 0$

which is the required equation of line through G and parallel to AC. **Ans.**

(c) L.H.S. $= \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$$= \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta\{1 - 2(1 - \cos^2\theta)\}}{\cos\theta(2\cos^2\theta - 1)}$$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta]$

$$= \frac{\sin\theta(1 - 2 + 2\cos^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta(2\cos^2\theta - 1)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta = \text{R.H.S.}$$

Hence Proved.

Solution 10.

(a) Let Vivek's present age be x years.

$\therefore$ His brother's present age = $(47 - x)$ years.

According to question,

$$x(47 - x) = 550$$

$\Rightarrow$ $47x - x^2 = 550$

$\Rightarrow$ $x^2 - 47x + 550 = 0$

$\Rightarrow$ $x^2 - 25x - 22x + 550 = 0$

$\Rightarrow$ $x(x - 25) - 22(x - 25) = 0$

$\Rightarrow$ $(x - 25)(x - 22) = 0$

$\Rightarrow$ $x - 25 = 0$ or $x - 22 = 0$

$\Rightarrow$ $x = 25$ or $x = 22$

When $x = 25$, $47 - x = 47 - 25 = 22$

When $x = 22$, $47 - x = 47 - 22 = 25$

(does not satisfy the given condition)

$\therefore$ Vivek's age = x = 25 years.

His younger brother's age = 22 years. **Ans.**

(b)

Wages (in ₹)	No. of Workers	Cumulative Frequency
400—450	2	2
450—500	6	8
500—550	12	20
550—600	18	38
600—650	24	62
650—700	13	75
700—750	5	80

$\therefore$ $n = 80$

(i) Median wage = $\frac{n}{2}$ th value.

$= \frac{80}{2}$ th value

= 40th value

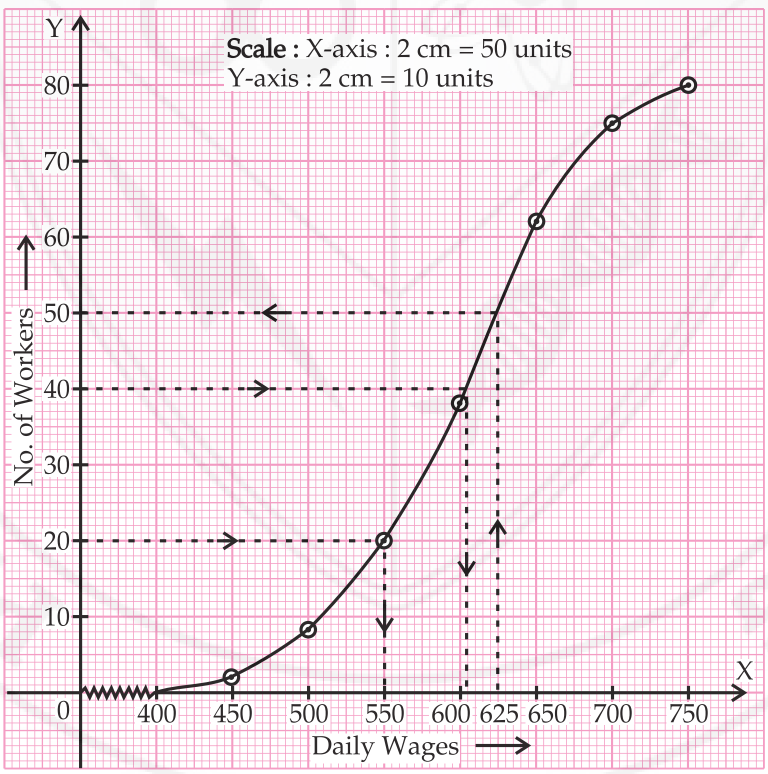
= ₹ 605 **Ans.**

(ii) Lower quartile = $\frac{n}{4}$ th value = 20th value

= ₹ 550 **Ans.**

(iii) No. of workers earning more than ₹ 625 daily

= 80 – 50 = 30 **Ans.**



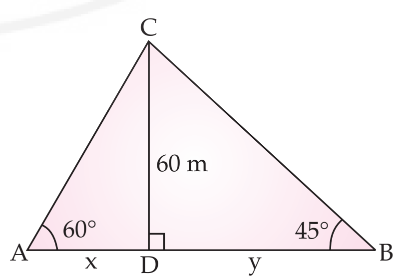
Solution 11.

(a) Let CD be the light house

$\therefore$ CD = 60 m.

Let AD = x m, BD = y m.

In Δ ACD,



$$\tan 60° = \frac{CD}{AD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{60}{x} \Rightarrow x = \frac{60}{\sqrt{3}}$$

$$\Rightarrow \quad x = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{60\sqrt{3}}{3}$$

$$= 20 \times 1{\cdot}732$$

$$= 34{\cdot}64 \text{ m}$$

In Δ BCD,

$$\tan 45° = \frac{CD}{BD}$$

$$1 = \frac{60}{y}$$

$$\Rightarrow \quad y = 60 \text{ m}$$

∴ Distance between two ships

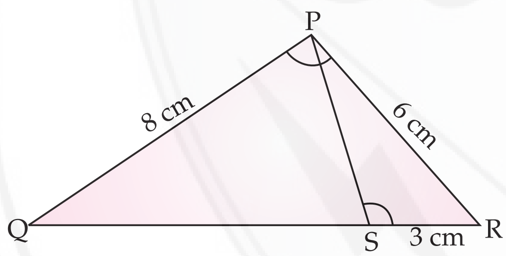
$$= x + y = 34{\cdot}64 + 60$$

$$= 94{\cdot}64 \text{ m}$$

$= 95$ m (correct to nearest whole number)

Ans.

(b) Given, ∠ PSR = ∠ QPR, QP = 8 cm, PR = 6 cm, SR = 3 cm.



(i) In Δ PQR and Δ SPR

∠ PSR = ∠ QPR (Given)

∠ R = ∠ R (Common angle)

∴ Δ PQR ~ Δ SPR (AA axiom)

Hence Proved.

Since, corresponding sides of similar Δs are proportional

(ii) $$\frac{PQ}{PS} = \frac{QR}{PR} = \frac{PR}{SR}$$

(∵ Δ PQR ~ Δ SPR)

$$\therefore \quad \frac{QR}{PR} = \frac{PR}{PS}$$

$$\Rightarrow \quad \frac{QR}{6} = \frac{6}{3}$$

$$\Rightarrow \quad QR = \frac{6 \times 6}{3} = 12 \text{ cm}$$ **Ans.**

Also, $$\frac{PQ}{PS} = \frac{PR}{SR}$$

$$\Rightarrow \quad \frac{8}{PS} = \frac{6}{3}$$

$$\Rightarrow \quad PS = \frac{8 \times 3}{6}$$

$$\Rightarrow \quad PS = 4 \text{ cm}$$ **Ans.**

(iii) $$\frac{\text{Area of } \Delta PQR}{\text{Area of } \Delta SPR} = \frac{PR^2}{SR^2}$$

$$= \frac{6^2}{3^2}$$

$$= \frac{36}{9} = \frac{4}{1} = 4 : 1$$ **Ans.**

(c) No. of months (n) = 3 × 12 = 36, R = 7·5%, Interest = ₹ 8325

(i) Let monthly deposit be ₹ x

$$\therefore \quad I = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$8325 = x \times \frac{36 \times 37}{2 \times 12} \times \frac{7.5}{100}$$

$$x = \frac{8325 \times 2 \times 100}{3 \times 37 \times 7.5}$$

$$\therefore \quad = 2000$$

∴ Monthly deposit is ₹ 2,000. **Ans.**

(ii) Total deposits = ₹ 2,000 × 36 = ₹ 72,000

∴ Maturity value = ₹ (72,000 + 8,325)

= ₹ 80,325 **Ans.**

MATHEMATICS 2016

QUESTIONS

SECTION—A (40 Marks)

*(Attempt **all** questions from this Section)*

Question 1.

(a) *Using remainder theorem, find the value of k if on dividing $2x^3 + 3x^2 - kx + 5$ by $x - 2$ leaves a remainder 7.* **[3]**

(b) *Given* $A = \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$ *and* $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ *and* $A^2 = 9A + mI$. *Find m.* **[4]**

(c) *The mean of following numbers is 68. Find the value of 'x'.* **[3]**

45, 52, 60, x, 69, 70, 26, 81 and 94.

Hence, estimate the median.

Question 2.

(a) *The slope of a line joining P (6, k) and Q (1– 3k, 3) is* $\frac{1}{2}$. *Find :* **[3]**

(i) *k*

(ii) *Midpoint of PQ, using the value of 'k' found in (i)*

(b) *Without using trigonometrical tables, evaluate :*** **[4]**

$\text{cosec}^2 57° - \tan^2 33° + \cos 44° \text{ cosec } 46° - \sqrt{2} \cos 45° - \tan^2 60°$

(c) *A certain number of metallic cones, each of radius 2 cm and height 3 cm are melted and recast into a solid sphere of radius 6 cm. Find the number of cones.* **[3]**

Question 3.

(a) *Solve the following inequation, write the solution set and represent it on the number line.* **[3]**

$$-3(x - 7) \geq 15 - 7x > \frac{x+1}{3}, x \in R$$

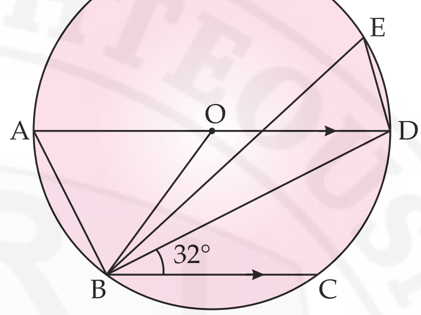
where R is a set of real numbers.

(b) *In the figure given below, AD is a diameter. O is the centre of the circle. AD is parallel to BC and ∠CBD= 32°. Find :* **[4]**

(i) *∠OBD*

(ii) *∠AOB*

(iii) *∠BED*



(c) *If (3a + 2b) : (5a + 3b) = 18 : 29. Find a : b.* **[3]**

Question 4.

(a) *A game of numbers has cards marked with 11, 12, 13, ……, 40. A card is drawn at random. Find the probability that the number on the card drawn is :* **[3]**

(i) *A perfect square*

(ii) *Divisible by 7*

(b) *Use graph paper for this question.* **[4]**

(Take 2 cm = 1 unit along both X and Y axis.)

Plot the points O (0, 0), A (– 4, 4), B (–3, 0) and C (0, – 3)

(i) *Reflect points A and B on the Y-axis and name them A′ and B′ respectively. Write down their coordinates.*

(ii) *Name the figure OABCB′A′.*

(iii) *State the line of symmetry of this figure.***

(c) *Mr. Lalit invested ₹ 5000 at a certain rate of interest, compounded annually for two years. At the end of first year it amounts to ₹ 5325. Calculate,* **[3]**

(i) *The rate of interest.*

(ii) *The amount at the end of second year, to the nearest rupee.***

SECTION—B (40 Marks)

*(Attempt any **four** questions from this Section)*

Question 5.

(a) *Solve the quadratic equation $x^2 - 3(x + 3) = 0$; Give your answer correct to two significant figures.* **[3]**

(b) *A page from the savings bank account of Mrs. Ravi is given below.*** **[4]**

** Answer is not given due to change in the present syllabus.

Date	Particulars	Withdrawal (₹)	Deposit (₹)	Balance (₹)
April 3rd, 2006	B/F			6000
April 7th	By Cash		2300	8300
April 15th	By Cheque		3500	11800
May 20th	To Self	4200		7600
June 10th	By Cash		5800	13400
June 15th	To Self	3100		10300
August 13th	By Cheque		1000	11300
August 25th	To Self	7400		3900
September 6th 2006	By Cash		2000	5900

She closed the account on 30th September, 2006. Calculate the interest Mrs. Ravi earned at the end of 30th September, 2006 at 4.5% per annum interest. Hence, find the amount she receives on closing the account.

(c) *In what time will ₹ 1500 yield ₹ 1996.50 as compound interest at 10% per annum compounded annually ?*** **[3]**

Question 6.

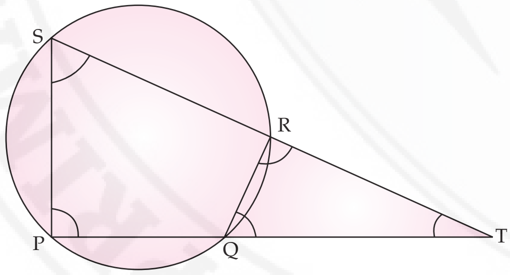
(a) *Construct a regular hexagon of side 5 cm. Hence construct all its lines of symmetry and name them.*** **[3]**

(b) *In the given figure PQRS is a cyclic quadrilateral PQ and SR produced meet at T.* **[4]**

(i) *Prove Δ TPS ~ Δ TRQ.*

(ii) *Find SP if TP = 18 cm, RQ = 4 cm and TR = 6 cm.*

(iii) *Find area of quadrilateral PQRS if area of Δ PTS = 27 cm^2.*



(c) *Given* $A = \begin{bmatrix} 4\sin 30° & \cos 0° \\ \cos 0° & 4\sin 30° \end{bmatrix}$ *and* $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ **[3]**

If AX = B

(i) *Write the order of matrix X.*

(ii) *Find the matrix 'X'.*

Question 7.

(a) *An aeroplane at an altitude of 1500 metres finds that two ships are sailing towards it in the same direction. The angles of depression as observed from the aeroplane are 45° and 30° respectively. Find the distance between the two ships.* **[4]**

** Answer is not given due to change in the present syllabus.

(b) *The table shows the distribution of the scores obtained by 160 shooters in a shooting competition. Use a graph sheet and draw an ogive for the distribution. (Take 2 cm = 10 scores on the X-axis and 2 cm = 20 shooters on the Y-axis)* **[6]**

Score	No. of Shooters
0–10	9
10–20	13
20–30	20
30–40	26
40–50	30
50–60	22
60–70	15
70–80	10
80–90	8
90–100	7

Use your graph to estimate the following :

(i) *The median.*

(ii) *The interquartile range.*

(iii) *The number of shooters who obtained a score of more than 85%.*

Question 8.

(a) *If* $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ *show that* $\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{3xyz}{abc}$ **[3]**

(b) *Draw a line AB = 5 cm. Mark a point C on AB such that AC = 3 cm. Using a ruler and a compass only, construct :* **[4]**

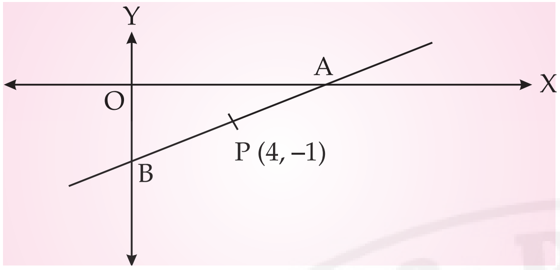
(i) *A circle of radius 2.5 cm, passing through A and C.*

(ii) *Construct two tangents to the circle from the external point B. Measure and record the length of the tangents.*

(c) *A line AB meets X-axis at A and Y-axis at B. P (4, –1) divides AB in the ratio 1 : 2.* **[3]**

(i) *Find the coordinates of A and B.*

(ii) *Find the equation of the line through P and perpendicular to AB.*



Question 9.

(a) *A dealer buys an article at a discount of 30% from the wholesaler, the marked price being ₹ 6,000. The dealer sells it to a shopkeeper at a discount of 10% on the marked price. If the rate of VAT is 6% find.*** **[3]**

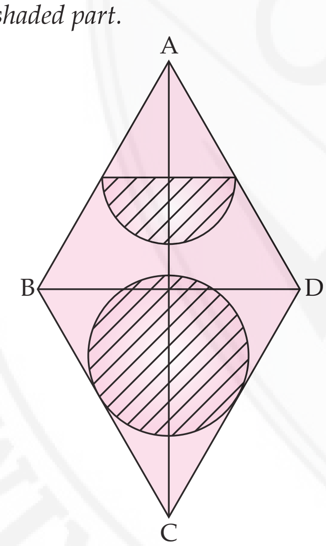
(i) *The price paid by the shopkeeper including the tax.*

(ii) *The VAT paid by the dealer.*

(b) *The given figure represents a kite with a circular and a semicircular motifs stuck on it. The radius of circle is 2.5 cm and the semicircle is 2 cm. If diagonals AC and BD are of lengths 12 cm and 8 cm respectively, find the area of the :*** **[4]**

(i) *shaded part. Give your answer correct to the nearest whole number.*

(ii) *unshaded part.*



(c) *A model of a ship is made to a scale 1 : 300* **[3]**

(i) *The length of the model of the ship is 2 m. Calculate the length of the ship.*

(ii) *The area of the deck ship is 180,000 m^2. Calculate the area of the deck of the model.*

(iii) *The volume of the model is 6.5 m^3. Calculate the volume of the ship.*

Question 10.

(a) *Mohan has a recurring deposit account in a bank for 2 years at 6% p.a. simple interest. If he gets ₹ 1200 as interest at the time of maturity, find :* **[3]**

(i) *the monthly instalment*

(ii) *the amount of maturity.*

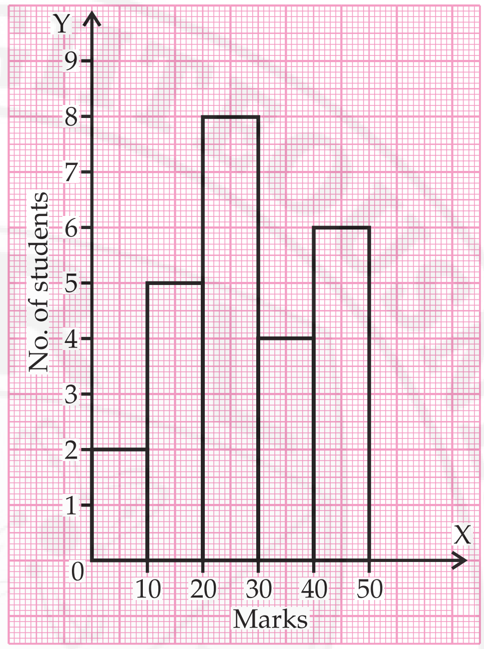
** Answer is not given due to change in the present syllabus.

(b) *The histogram below represents the scores obtained by 25 students in a Mathematics mental test. Use the data to :* **[4]**

(i) *Frame a frequency distribution table.*

(ii) *To calculate mean.*

(iii) *To determine the Modal class.*



(c) *A bus covers a distance of 240 km at a uniform speed. Due to heavy rain its speed gets reduced by 10 km/h and as such it takes two hrs longer to cover the total distance. Assuming the uniform speed to be 'x' km/h, form an equation and solve it to evaluate 'x'.* **[3]**

Question 11.

(a) *Prove that* $\frac{\cos A}{1+\sin A} + \tan A = \sec A$. **[3]**

(b) *Use ruler and compasses only for the following question. All construction lines and arcs must be clearly shown.* **[4]**

(i) *Construct a Δ ABC in which BC = 6.5 cm, ∠ABC = 60°, AB = 5 cm.*

(ii) *Construct the locus of points at a distance of 3.5 cm from A.*

(iii) *Construct the locus of points equidistant from AC and BC.*

(iv) *Mark 2 points X and Y which are at a distance of 3.5 cm from A and also equidistant from AC and BC. Measure XY.*

(c) *Ashok invested ₹ 26,400 on 12%, ₹ 25 shares of a company. If he receives a dividend of ₹ 2,475, find the:* **[3]**

(i) *number of shares he bought*

(ii) *Market value of each share*

Answers

SECTION—A

Solution 1.

(a) Here, $f(x) = 2x^3 + 3x^2 - kx + 5$ ...(i)

and $x - 2 = 0 \Rightarrow x = 2$

Given, remainder is $7 \Rightarrow f(2) = 7$

Putting $x = 2$ in equation (i), we get

$$f(2) = 2(2)^3 + 3(2)^2 - k(2) + 5$$

$\Rightarrow$ $16 + 12 - 2k + 5 = 7$

$\Rightarrow$ $2k = 26$

$\Rightarrow$ $k = 13.$ **Ans.**

(b) Here, $A = \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$

$$\Rightarrow A^2 = A \cdot A = \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 0+0 \\ -2-7 & 0+49 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix}$$

Given, $A^2 = 9A + mI$

$$\therefore 9\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix} + m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 18 & 0 \\ -9 & 63 \end{bmatrix} + \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 18+m & 0 \\ -9 & 63+m \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix}$$

On comparing both sides, we get

$$18 + m = 4 \text{ and } 63 + m = 49$$

which gives $m = -14$. **Ans.**

(c) Arithmetic mean $= \frac{\Sigma x}{n}$

$$= \frac{45 + 52 + 60 + x + 69 + 70 + 26 + 81 + 94}{9}$$

$\Rightarrow$ $68 = \frac{497 + x}{9}$

$\Rightarrow$ $612 = 497 + x$

$\Rightarrow$ $x = 612 - 497$

$\Rightarrow$ $x = 115$

On arranging the given terms in ascending order of magnitude, we get

26, 45, 52, 60, 69, 70, 81, 94, 115

Since number of terms are odd

$$\therefore \text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{term}$$

$$= \left(\frac{9+1}{2}\right)^{th} \text{term}$$

$$= 5^{th} \text{ term} = 69$$

So, the median is 69. **Ans.**

Solution 2.

(a) **(i)** Let P $(6, k)$ be (x_1, y_1) and Q $(1 - 3k, 3)$ be (x_2, y_2)

Given, slope of a line PQ is $\frac{1}{2}$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1}{2}$$

Here, $x_1 = 6, x_2 = 1 - 3k, y_1 = k, y_2 = 3$

$\Rightarrow$ $\frac{3-k}{(1-3k)-6} = \frac{1}{2}$

$\Rightarrow$ $\frac{3-k}{-5-3k} = \frac{1}{2}$

$\Rightarrow$ $2(3-k) = -5-3k$

$\Rightarrow$ $6 - 2k = -5 - 3k$

$\Rightarrow$ $k = -11$ **Ans.**

(ii) Coordinates P is $(6, -11)$ and Q is $(34, 3)$

Midpoint of P $(6, -11)$ and Q $(34, 3)$.

$\Rightarrow$ $\left(\frac{6+34}{2}, \frac{-11+3}{2}\right) = (20, -4)$ **Ans.**

(c) Volume of metallic cone

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \times (2)^2 \times 3 = 4\pi \text{ cm}^3$$

Volume of solid sphere

$$= \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi \times (6)^3$$

$$= 288\pi \text{ cm}^3$$

$\therefore$ Number of cones

$$= \frac{\text{Volume of solid sphere}}{\text{Volume of metallic cone}}$$

$$= \frac{288\pi}{4\pi} = 72$$

$\therefore$ Number of metallic cones are 72. **Ans.**

Solution 3.

(a) Given, $-3(x-7) \geq 15 - 7x > \frac{x+1}{3}$

$\Rightarrow -3(x-7) \geq 15 - 7x$ and $\Rightarrow 15 - 7x > \frac{x+1}{3}$

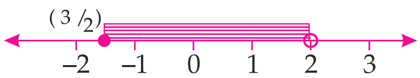
$\Rightarrow -3x + 21 \geq 15 - 7x$ and $\Rightarrow 45 - 21x > x + 1$

$\Rightarrow 7x - 3x \geq 15 - 21$ and $\Rightarrow -21x - x > 1 - 45$

$\Rightarrow 4x \geq -6$ and $\Rightarrow -22x > -44$

$\Rightarrow x \geq -\frac{3}{2}$ and $\Rightarrow x < 2$

On simplifying, the given inequation reduces to $-\frac{3}{2} \leq x < 2$ and the required number line is



Solution 4. set is $\left\{x : -\frac{3}{2} \leq x < 2, x \in R\right\}$ **Ans.**

(b) **(i)** Since, AD is parallel to BC and BD is a transversal.

$\therefore$ $\angle ODB = \angle CBD$

(Alternate interior angles)

Also, OB = OD (Radii)

$\therefore$ $\angle OBD = \angle ODB = 32°$ **Ans.**

(ii) $\angle AOB = 2\angle ADB$

(The angle that an arc of a circle subtends at the centre is double which it subtends at any point on the remaining part of the circle)

$= 2 \times 32°$

$= 64°$ **Ans.**

(iii) In Δ AOB (OA = OB, radii of circle)

$\therefore$ $\angle OAB + \angle AOB + \angle OBA = 180°$

$\angle OAB + 64° + \angle OAB = 180°$

$2\angle OAB = 180° - 64°$

$= 116°$

$\angle OAB = 58°$

$\angle OAB = \angle BED = 58°$

(Angles in the same segment)

Ans.

(c) Here, $\frac{3a+2b}{5a+3b} = \frac{18}{29}$

$87a + 58b = 90a + 54b$

$-90a + 87a = -58b + 54b$

$-3a = -4b$

$\frac{a}{b} = \frac{4}{3}$

i.e., $a : b = 4 : 3$ **Ans.**

Solution 4.

(a) The possible outcomes are 11, 12, 13 ……, 40. Total number of all possible outcomes *i.e.,* $n(S) = 30$

(i) For getting a perfect square :

The favourable outcomes are : 16, 25, 36

No. of favourable outcomes n (A) = 3

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{30} = \frac{1}{10}$$ **Ans.**

(ii) For getting a number divisible by 7 :

The favourable outcomes are : 14, 21, 28, 35.

No. of favourable outcomes, n (B) = 4

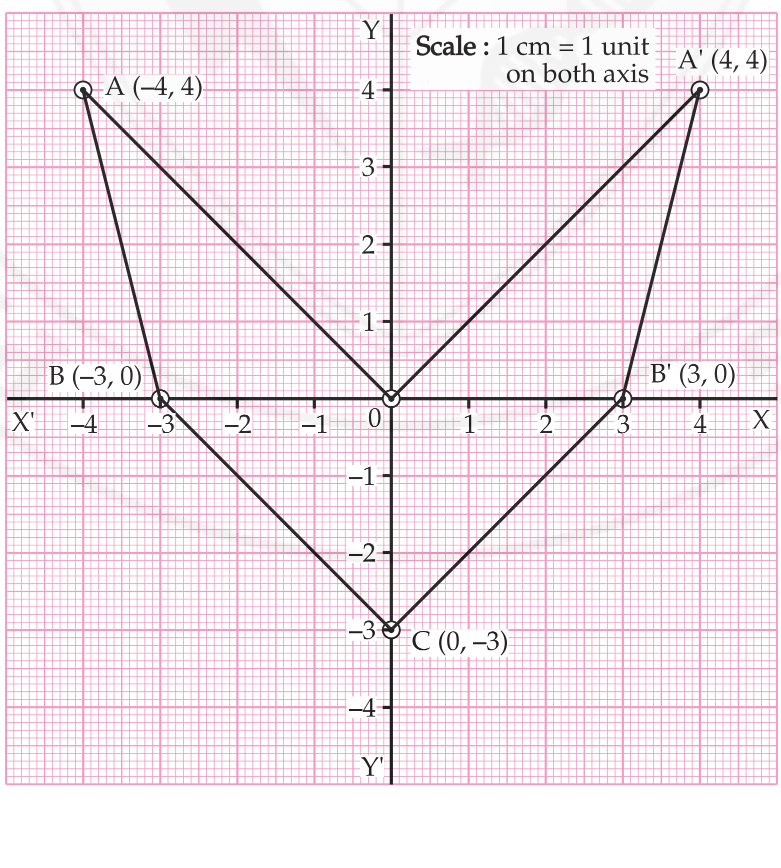
Required probability

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{30} = \frac{2}{15}$$ **Ans.**

(b) **(i)** Coordinates of A′ = (4, 4)

Coordinates of B′ = (3, 0)

(ii) Irregular Hexagon or Concave region



SECTION—B

Solution 5.

(a) Given, $x^2 - 3(x+3) = 0$

$x^2 - 3x - 9 = 0$

Compare it with $ax^2 + bx + c = 0$, we get

$a = 1, b = -3$ and $c = -9$

$\therefore \quad D = b^2 - 4ac = (-3)^2 - 4 \times 1 \times (-9)$

$= 9 + 36 = 45$

$\therefore \quad x = \frac{3 \pm \sqrt{45}}{2 \times 1}$

$\left[\because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$

$\Rightarrow \quad x = \frac{3 \pm 6.708}{2}$

$\Rightarrow \quad x = \frac{3 + 6.708}{2}$ and $\frac{3 - 6.708}{2}$

$\Rightarrow \quad x = 4{\cdot}854$ and $-1{\cdot}854$

The roots of given equation are 4.85 and –1.85. **Ans.**

Solution 6.

(b) (i) In Δ TPS and Δ TRQ

$\angle STP = \angle QTR$ (common)

$\angle TPS = \angle TRQ$

($\because$ Exterior angle of cyclic quadrilateral = Interior opposite angle)

$\therefore \quad \Delta TPS \sim \Delta TRQ$ [By AA similarity]

Hence Proved.

(ii) Since $\Delta TPS \sim \Delta TRQ$

$\therefore \quad \frac{TR}{TP} = \frac{RQ}{SP}$

(corresponding sides of similar Ds are proportional)

$\frac{6}{18} = \frac{4}{SP}$

$SP = \frac{4 \times 18}{6} = 12$ cm **Ans.**

(iii) We know that the ratio between the areas of two similar triangles is equal to the ratio between the squares of its corresponding sides.

$\frac{ar(\Delta PTS)}{ar(\Delta RTQ)} = \frac{(SP)^2}{(QR)^2} = \frac{(12)^2}{(4)^2} = \frac{9}{1}$

$\frac{27}{ar(\Delta RTQ)} = \frac{9}{1}$

$ar(\Delta RTQ) = 3\ cm^2$

$\therefore$ ar (quadrilateral PQRS)

$= ar(\Delta PTS) - ar(\Delta RTQ)$

$= 27 - 3 = 24\ cm^2$ **Ans.**

(c) (i) Given, $A = \begin{bmatrix} 4\sin 30° & \cos 0° \\ \cos 0° & 4\sin 30° \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$A = \begin{bmatrix} 4 \times \frac{1}{2} & 1 \\ 1 & 4 \times \frac{1}{2} \end{bmatrix}$

$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$\because$ The order of matrix A is 2×2

and the order of matrix B is 2×1

$\therefore$ The order of matrix X is 2×1. **Ans.**

(ii) Let $X = \begin{bmatrix} x \\ y \end{bmatrix}$

$\therefore \quad \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 2x + y \\ x + 2y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

On comparing, we get

$2x + y = 4$ ...(i)

and $x + 2y = 5$ ...(ii)

On multiplying equation (ii) by 2 and subtracting it from equation (i)

$2x + y = 4$

$2x + 4y = 10$

$- \quad - \quad -$

$-3y = -6$

$y = 2$

From equation (i)

$2x + y = 4$

$\Rightarrow \quad 2x + 2 = 4$

$x = 1$

$\therefore$ The matrix $X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ **Ans.**

Solution 7.

(a) Let AB be the altitude and C and D be the positions of two ships.

In right angled triangle ABC,

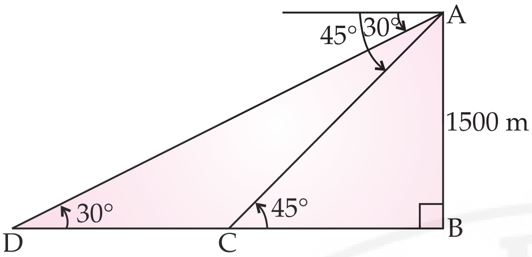
$\tan 45° = \frac{1500}{BC}$

$1 = \frac{1500}{BC}$

$BC = 1500$ m

In right angled triangle ABD,

$\tan 30° = \frac{1500}{BD}$



$$\frac{1}{\sqrt{3}} = \frac{1500}{BD}$$

$\Rightarrow$ $BD = 1500\sqrt{3}$

$= 1500 \times 1{\cdot}732$

$= 2598$ m

$\therefore$ Distance between the two ships

$= CD$

$= BD - BC$

$= 2598 - 1500$

$= 1098$ m **Ans.**

(b)

Scores	No. of Shooters	Cumulative frequency (c. f.)
0—10	9	9
10—20	13	22
20—30	20	42
30—40	26	68
40—50	30	98
50—60	22	120
60—70	15	135
70—80	10	145
80—90	8	153
90—100	7	160

Using graph = 160

(i) Since, $n = 160$ (even)

$$\text{Median} = \left(\frac{n}{2}\right)^{th} \text{term}$$

$$= \left(\frac{160}{2}\right)^{th} \text{term}$$

$= 80$th term

$= 44$ **Ans.**

(ii) Lower quartile

$$(Q_1) = \left(\frac{n}{4}\right)^{th} \text{term}$$

$$= \left(\frac{160}{4}\right)^{th} = 40^{th} \text{ term.}$$

$= 29$

Upper quartile

$$(Q_3) = \left(\frac{3n}{4}\right)^{th} \text{term}$$

$$= \left(\frac{3 \times 160}{4}\right)^{th} \text{term}$$

$= 120^{th}$ term.

$= 60$

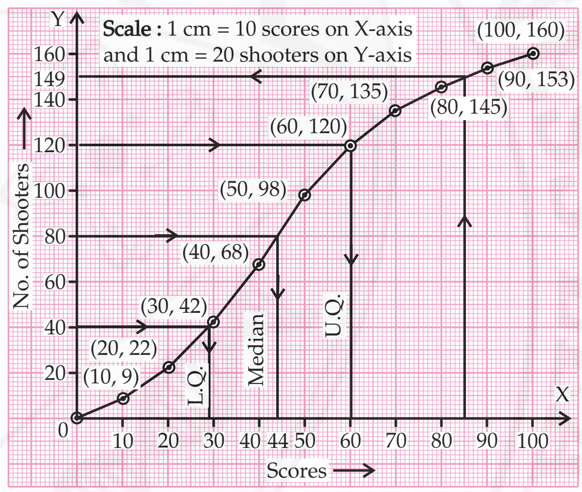
Inter-quartile range

$= Q_3 - Q_1$

$= 60 - 29 = 31$ **Ans.**

(iii) Since, 85% scores = 85% of 100 = 85.

Through mark for 85 on X-axis, draw a vertical line which meets the ogive at any point. Through that point, draw a horizontal line which meets the Y-axis at the mark of 149.



$\therefore$ The no. of shooters who obtained a score of more than 85%

$= 160 - 149 = 11$ **Ans.**

Note : Instead of 2 cm = 1 unit, we have taken 1 cm = 1 unit both axes.

Solution 8.

(a) Let $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = k$

then $x = ak$, $y = bk$ and $z = ck$.

Putting the values of x, y and z, in the given equation, we get

$$\text{L.H.S.} = \frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3}$$

$$= \frac{(ak)^3}{a^3} + \frac{(bk)^3}{b^3} + \frac{(ck)^3}{c^3}$$

$$= \frac{a^3k^3}{a^3} + \frac{b^3k^3}{b^3} + \frac{c^3k^3}{c^3}$$

$$= k^3 + k^3 + k^3 = 3k^3$$

R.H.S. $= \frac{3xyz}{abc}$

(Put the value of x, y and z)

$= \frac{3(ak)(bk)(ck)}{abc}$

$= 3k^3$

$\therefore$ L.H.S. = R.H.S. **Hence Proved.**

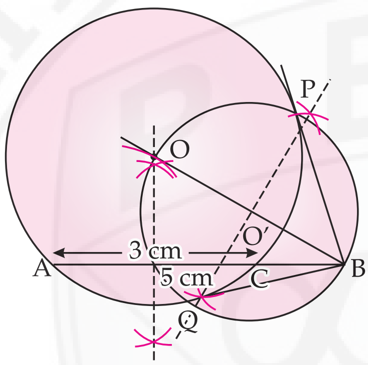
(b) **(i)** Steps of construction :

1. Draw a line AB = 5 cm.

2. Mark a point C on AB such that AC = 3 cm.

3. Draw a perpendicular bisector of AC.

4. Mark a point O on perpendicular bisector from A of length 2·5 cm.



5. Taking O as centre and OA as radius draw a circle, which is the required circle.

(ii) 1. Join O and B.

2. Draw a circle with OB as diameter which cuts the given circle at points P and Q. For this draw perpendicular bisector of OB which cuts OB at O'. Draw a circle with OO' as radius.

3. This circle with centre O' cut the other circle at P and Q.

4. Join PB and QB, which are the required tangents.

QB = PB = 3 cm

Hence, length of tangents is 3 cm.

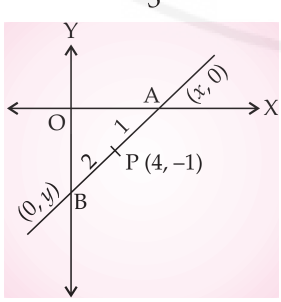
(c) **(i)** Let the coordinates of A be $(x, 0)$ and B be $(0, y)$.

Given, P = (4, – 1) divides AB in the ratio 1 : 2.

Now, $x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$

$4 = \frac{1 \times 0 + 2 \times x}{1+2}$

$4 = \frac{2x}{3}$



$\therefore \quad x = 6$

and $\quad y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$

$-1 = \frac{1 \times y + 2 \times 0}{1+2}$

$-1 = \frac{y}{3}$

$\therefore \quad y = -3$

$\therefore$ Coordinates of A are (6, 0) and coordinates of B are (0, – 3). **Ans.**

(ii) Slope of AB $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-0}{0-6} = \frac{-3}{-6} = \frac{1}{2}$

Now, slope of the line perpendicular to AB

$= -\frac{1}{\text{slope of AB}}$

$= -\frac{1}{1/2} = -2$

Equation of line, which passes through P (4, – 1), and ⊥ to AB and has slope – 2 is

$y - y_1 = m(x - x_1)$

$y - (-1) = -2(x - 4)$

$y + 1 = -2x + 8$

Hence, $\quad 2x + y = 7$ **Ans.**

Solution 9.

(c) Given, scale factor $(k) = \frac{1}{300}$

(i) $\frac{\text{Length of the model}}{\text{Length of the ship}} = k = \frac{1}{300}$

$\Rightarrow \quad \frac{2}{\text{Length of ship}} = \frac{1}{300}$

Length of the ship = 2 × 300 = 600 m **Ans.**

(ii) $\frac{\text{Area of the deck of the model}}{\text{Area of the deck of the ship}} = k^2$

$\frac{\text{Area of deck of model}}{1{,}80{,}000} = \frac{1 \times 1}{300 \times 300}$

Area of the deck of the model

$= \frac{180{,}000}{300 \times 300} = 2 \text{ m}^2$ **Ans.**

(iii) $\frac{\text{Volume of model}}{\text{Volume of the ship}} = k^3$

$\Rightarrow \quad \frac{6.5}{\text{Volume of ship}} = \frac{1 \times 1 \times 1}{300 \times 300 \times 300}$

Volume of the ship = 6·5 × 300 × 300 × 300

= 17,55,00,000 m^3 **Ans.**

Solution 10.

(a) **(i)** Given, number of months (n) = 24 and rate of interest (r) = 6% and Interest = ₹ 1,200

$$I = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$1200 = P \times \frac{24(24+1)}{2 \times 12} \times \frac{6}{100}$$

$$P = \frac{1,200 \times 24 \times 100}{6 \times 24 \times 25}$$

$$= ₹\ 800$$

∴ Monthly instalment = ₹ 800 **Ans.**

(ii) Sum deposited = ₹ 800 × 24

= ₹ 19,200

Amount of maturity = ₹ 19,200 + ₹ 1,200

= ₹ 20,400 **Ans.**

(b) **(i)** Using the given data, frequency distribution table is as given below :

Marks	No. of Students (f)	Class mark (x)	fx
0—10	2	5	10
10—20	5	15	75
20—30	8	25	200
30—40	4	35	140
40—50	6	45	270
	$\Sigma f = n = 25$		$\Sigma f.x = 695$

(ii) To Calculate Mean : Construct expanded table with class mark and fx as given above.

∴ $n = \Sigma f = 25$ and $\Sigma fx = 695$

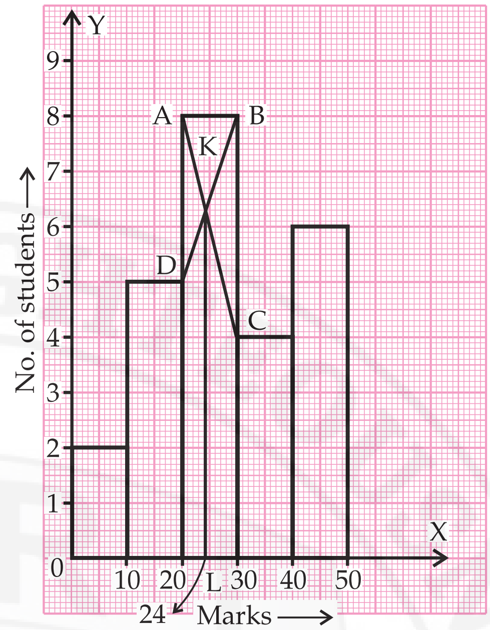
$$\text{Mean} = \frac{\Sigma fx}{n} = \frac{695}{25} = 27.8$$ **Ans.**

(iii) 1. In the given histogram, inside the highest rectangle, which represents the maximum frequency (or modal class) draw two lines AC and BD diagonally from the upper corners to C and D of adjacent rectangles.

2. Both the lines meet at a point K. Through the point K, draw KL perpendicular to the horizontal axis.

3. The value of point L on the horizontal axis represents the value of mode.

∴ Mode = 24 and the modal class = 20 – 30.



(c) Let the uniform speed of bus be x km/h.

∴ Time taken by it to cover 240 km = $\frac{240}{x}$ hrs.

$$\left[\because T = \frac{D}{S}\right]$$

Reduced speed of bus = $(x - 10)$ km/hr

∴ So, time taken by the bus to cover 240 km

$$= \frac{240}{x-10} \text{ hrs.}$$

Now, according to the given condition

$$\therefore \quad \frac{240}{x-10} - \frac{240}{x} = 2$$

$$\Rightarrow \quad 240\left(\frac{1}{x-10} - \frac{1}{x}\right) = 2$$

$$\Rightarrow \quad 120\left(\frac{x-(x-10)}{x(x-10)}\right) = 1$$

$$\Rightarrow \quad 120\left(\frac{10}{x(x-10)}\right) = 1$$

$$\Rightarrow \quad x^2 - 10x - 1200 = 0$$

On splitting the middle term

$$\Rightarrow \quad x^2 - 40x + 30x - 1200 = 0$$

$$\Rightarrow \quad x(x-40) + 30(x-40) = 0$$

$$\Rightarrow \quad (x-40)(x+30) = 0$$

$$\Rightarrow \quad x = 40$$

or $\quad x = -30$

Since, speed cannot be negative.

Hence, the value of $x = 40$.

i.e., the uniform speed of bus is 40 km/hr. **Ans.**

Solution 11.

(a) L.H.S. $= \dfrac{\cos A}{1+\sin A} + \tan A$

$= \dfrac{\cos A}{1+\sin A} + \dfrac{\sin A}{\cos A}$ $\left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right]$

$= \dfrac{\cos^2 A + \sin A + \sin^2 A}{(1+\sin A)\cos A}$

$= \dfrac{1+\sin A}{(1+\sin A)\cos A}$

$[\because \cos^2\theta + \sin^2\theta = 1]$

$= \dfrac{1}{\cos A}$ = sec A = R.H.S.

Hence Proved.

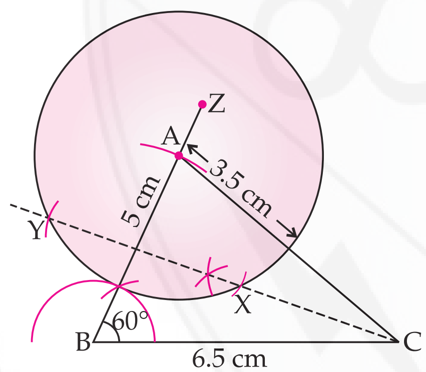
(b) Steps of Construction :

(i) 1. Draw a line BC = 6·5 cm.

2. At B, draw BZ making an angle of 60° with BC.

3. With B as centre, draw an arc of 5 cm. It cuts BZ at point A.

4. Join AC.



(ii) Taking A as centre and 3·5 cm as radius, draw a circle which is the required locus of points.

(iii) Draw the bisector of angle of vertex ACB, which is the required locus of points equidistant from AC and BC.

From A cut the arcs of length 3.5 cm on the line which is the angle bisector of $\angle$ BCA and touch the circle with centre A. The required points are X and Y.

(iv) Length of XY = 5 cm.

(c) **(i)** Given, nominal value of share = ₹ 25, Rate of dividend = 12%

Total dividend = ₹ 2,475

Total money invested = ₹ 26,400

Dividend on each share

= Rate of dividend × N. V.

$= \dfrac{12}{100} \times 25 =$ ₹ 3

No. of shares bought

$= \dfrac{\text{Total dividend}}{\text{Dividend on each share}}$

$= \dfrac{2475}{3}$

= 825 **Ans.**

(ii) Market value of each share

$= \dfrac{\text{Sum invested}}{\text{No. of shares}}$

$= \dfrac{26,400}{825}$

= ₹ 32. **Ans.**

MATHEMATICS 2015

QUESTIONS

SECTION—A (40 Marks)

*(Attempt **all** questions from this Section)*

Question 1.

(a) *A shopkeeper bought an article for ₹ 3,450. He marks the price of the article 16% above the cost price. The rate of sales tax charged on the article is 10%. Find the :***

(i) *marked price of the article.*

(ii) *price paid by a customer who buys the article.* **[3]**

(b) *Solve the following inequation and write the solution set :*

$$13x - 5 < 15x + 4 < 7x + 12, x \in R$$

Represent the solution on a real number line. **[3]**

(c) *Without using trigonometric tables evaluate :*** **[4]**

$$\frac{\sin 65^\circ}{\cos 25^\circ} + \frac{\cos 32^\circ}{\sin 58^\circ} - \sin 28^\circ . \sec 62^\circ + \text{cosec}^2 30^\circ$$

Question 2.

(a) *If* $A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$ *and* $B = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$, *find x and y when* $A^2 = B$. **[3]**

(b) *The present population of a town is 2,00,000. Its population increases by 10% in the first year and 15% in the second year. Find the population of the town at the end of the two years.*** **[3]**

(c) *Three vertices of a parallelogram ABCD taken in order are A (3, 6), B (5, 10) and C (3, 2) find :*

(i) *the coordinates of the fourth vertex D.*

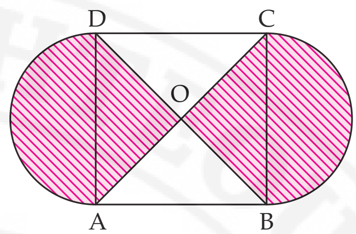
(ii) *length of diagonal BD.***

(iii) *equation of side AB of the parallelogram ABCD.* **[4]**

Question 3.

(a) *In the given figure, ABCD is a square of side 21 cm. AC and BD are two diagonals of the square. Two semi circles are drawn with AD and BC as diameters. Find the area of the shaded region.*** $\left(\text{Take } \pi = \frac{22}{7}\right)$ **[3]**

** Answer is not given due to change in the present syllabus.

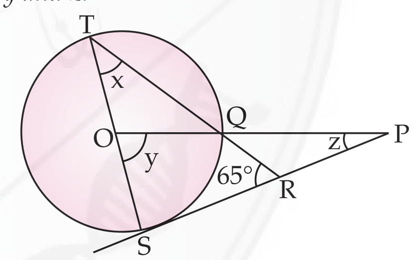


(b) *The marks obtained by 30 students in a class assessment of 5 subjects is given below :*

Marks	0	1	2	3	4	5
No. of Students	1	3	6	10	5	5

Calculate the mean, median and mode of the above distribution. **[3]**

(c) *In the figure given below, O is the centre of the circle and SP is a tangent. If ∠SRT = 65°, find the value of x, y and z.* **[4]**



Question 4.

(a) *Katrina opened a recurring deposit account with a Nationalised Bank for a period of 2 years. If the bank pays interest at the rate of 6% per annum and the monthly instalment is ₹ 1,000, find the :*

(i) *interest earned in 2 years.*

(ii) *maturity value.* **[3]**

(b) *Find the value of 'K' for which x = 3 is a solution of the quadratic equation,*

$(K + 2)x^2 - Kx + 6 = 0.$

Thus, find the other root of the equation. **[3]**

(c) *Construct a regular hexagon of side 5 cm. Construct a circle circumscribing the hexagon. All traces of construction must be clearly shown.* **[4]**

SECTION—B (40 Marks)

*(Attempt any **four** questions from this Section)*

Question 5.

(a) *Use a graph paper for this question take 1 cm = 1 unit along both the X and Y axis :*

ICSE Mathematics-2015

(i) *Plot the points A(0, 5), B(2, 5), C(5, 2), D(5, –2), E(2, –5) and F(0, –5).*

(ii) *Reflect the points B, C, D and E on the Y-axis and name them respectively as B′, C′, D′ and E′.*

(iii) *Write the coordinates of B′, C′, D′ and E′.*

(iv) *Name the figure formed by BCDEE′D′C′B′.*

(v) *Name a line of symmetry for the figure formed.* ** **[5]**

(b) *Virat opened a Savings Bank account in a bank on 16 th April, 2010. His pass book shows the following entries :* **

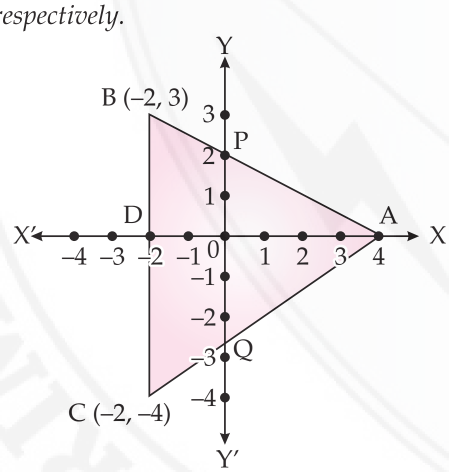
Date	Particulars	Withdrawal (₹)	Deposit (₹)	Balance (₹)
April 16, 2010	By Cash	—	2500	2500
April 28th	By Cheque	—	3000	5500
May 9th	To Cheque	850	—	4650
May 15th	By Cash	—	1600	6250
May 24th	To Cash	1000	—	5250
June 4th	To Cash	500	—	4750
June 30th	By Cheque	—	2400	7150
July 3rd	By Cash	—	1800	8950

Calculate the interest Virat earned at the end of 31st July, 2010 at 4% per annum interest. What sum of money will he receive if he closes the account on 1st August, 2010 ? **[5]**

Question 6.

(a) *If a, b, c are in continued proportion, prove that* $(a + b + c)(a - b + c) = a^2 + b^2 + c^2$. **[3]**

(b) *In the given figure ABC is a triangle and BC is parallel to the Y-axis. AB and AC intersects the y-axis at P and Q respectively.*



(i) *Write the coordinates of A.*

(ii) *Find the length of AB and AC.* **

(iii) *Find the ratio in which Q divides AC.*

(iv) *Find the equation of the line AC.* **[4]**

(c) *Calculate the mean of the following distribution :* **[3]**

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	8	5	12	35	24	16

Question 7.

(a) *Two solid spheres of radii 2 cm and 4 cm are melted and recast into a cone of height 8 cm. Find the radius of the cone so formed.* **[3]**

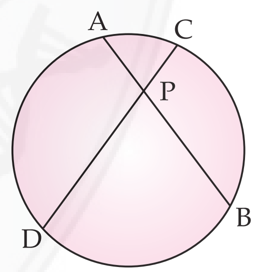
** Answer is not given due to change in the present syllabus.

(b) *Find 'a' if the two polynomials* $ax^3 + 3x^2 - 9$ *and* $2x^3 + 4x + a$*, leaves the same remainder when divided by* $x + 3$. **[3]**

(c) *Prove that* $\dfrac{\sin\theta}{1 - \cot\theta} + \dfrac{\cos\theta}{1 - \tan\theta} = \cos\theta + \sin\theta$ **[4]**

Question 8.

(a) *AB and CD are two chords of a circle intersecting at P.*
Prove that AP × PB = CP × PD **[3]**



(b) *A bag contains 5 white balls, 6 red balls and 9 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is :*

(i) *a green ball*

(ii) *a white or red ball*

(iii) *is neither a green ball nor a white ball.* **[3]**

(c) *Rohit invested ₹ 9,600 on ₹ 100 shares at ₹ 20 premium paying 8% dividend. Rohit sold the shares when the price rose to ₹ 160. He invested the proceeds (excluding dividend) in 10% ₹ 50 shares at ₹ 40. Find the :*

(i) *original number of shares.*

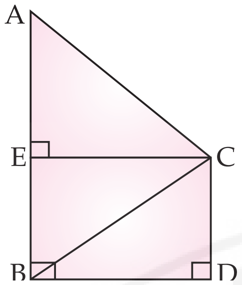
(ii) *sale proceeds.*

(iii) *new number of shares.*

(iv) *change in the two dividends.* **[4]**

Question 9.

(a) *The horizontal distance between two towers is 120 m. The angle of elevation of the top and angle of depression of the bottom of the first tower as observed from the second tower is 30° and 24° respectively.*



Find the height of the two towers. Give your answer correct to 3 significant figures. **[4]**

(b) *The weight of 50 workers is given below:*

Weight in kg	50-60	60-70	70-80	80-90	90-100	100-110	110-120
No. of Workers	4	7	11	14	6	5	3

Draw an ogive of the given distribution using a graph sheet. Take 2 cm = 10 kg on one axis and 2 cm = 5 workers along the other axis. Use a graph to estimate the following :

(i) *the upper and lower quartiles.*

(ii) *if weight 95 kg and above is considered overweight find the number of workers who are overweight.* **[6]**

Question 10.

(a) *A wholesaler buys a TV from the manufacturer for ₹ 25,000. He marks the price of the TV 20% above his cost price and sells it to a retailer at a 10% discount on the marked price. If the rate of VAT is 8%, Find the :***

(i) *marked price.*

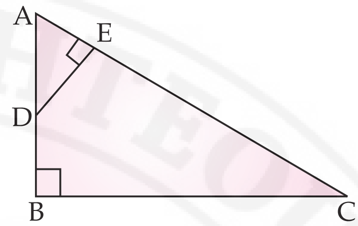
(ii) *retailer's cost price inclusive of tax.*

(iii) *VAT paid by the wholesaler.* **[3]**

(b) *If* $A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ *and* $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$

Find AB – 5C. **[3]**

(c) *ABC is a right angled triangle with ∠ABC = 90°, D is any point on AB and DE is perpendicular to AC. Prove that :*



(i) *Δ ADE ~ Δ ACB.*

(ii) *If AC = 13 cm, BC = 5 cm and AE = 4 cm. Find DE and AD.*

(iii) *Find area of Δ ADE : area of quadrilateral BCED.* **[4]**

Question 11.

(a) *Sum of two natural numbers is 8 and the difference of their reciprocal is* $\frac{2}{15}$. *Find the numbers.* **[3]**

(b) *Given* $\frac{x^3+12x}{6x^2+8} = \frac{y^3+27y}{9y^2+27}$. *Using componendo and dividendo find* $x : y$. **[3]**

(c) *Construct a triangle ABC with AB = 5·5 cm, AC = 6 cm and ∠BAC = 105°. Hence :*

(i) *Construct the locus of points equidistant from BA and BC.*

(ii) *Construct the locus of points equidistant from B and C.*

(iii) *Mark the point which satisfies the above two loci as P. Measure and write the length of PC.* **[4]**

ANSWERS

SECTION—A

Solution 1.

(b) Given, $13x - 5 < 15x + 4 < 7x + 12$

$\Rightarrow \quad 13x - 5 < 15x + 4$ and $15x + 4 < 7x + 12$

$\Rightarrow \quad 13x - 15x < 4 + 5$ and $15x - 7x < 12 - 4$

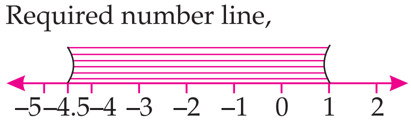
$\Rightarrow \quad -2x < 9$ and $8x < 8$

$\Rightarrow \quad -x < \frac{9}{2}$ and $x < 1$

$\Rightarrow \quad x > -4{\cdot}5$

$\therefore$ Solution set $= \{x : -4{\cdot}5 < x < 1 \text{ and } x \in R\}$

Required number line,



Ans.

** Answer is not given due to change in the present syllabus.

Solution 2.

(a) Here, $A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$

$$A^2 = A \cdot A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9+0 & 3x+x \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix}$$

According to the given condition,

$$A^2 = B$$

$$\begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$$

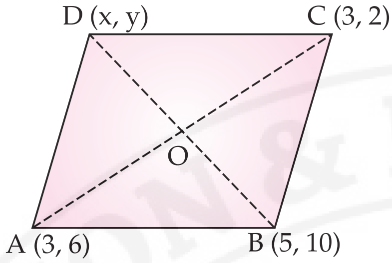
On comparing, we get,

$4x = 16$ and $-y = 1$

$\therefore \quad x = 4$ and $y = -1$. **Ans.**

(c) (i) Let the coordinates of the fourth vertex of a parallelogram be D (x, y).

Since, diagonals of a parallelogram bisects each other



∴ Mid point of AC = Mid point of BD

$$\Rightarrow \left(\frac{3+3}{2}, \frac{6+2}{2}\right) = \left(\frac{5+x}{2}, \frac{10+y}{2}\right)$$

$$\Rightarrow \quad (3, 4) = \left(\frac{5+x}{2}, \frac{10+y}{2}\right)$$

$$\Rightarrow \quad \frac{5+x}{2} = 3 \text{ and } \frac{10+y}{2} = 4$$

$$\Rightarrow \quad 5+x = 6 \text{ and } 10+y = 8$$

$$\Rightarrow \quad x = 1 \text{ and } y = -2$$

∴ The coordinates of the fourth vertex D is (1, – 2). **Ans.**

(iii) Here, A = (3, 6) and B (5, 10)

i.e., $x_1 = 3, x_2 = 5$

$y_1 = 6, y_2 = 10,$

$$\therefore \quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-6}{5-3} = 2$$

∴ Equation of side AB of the parallelogram ABCD is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - 6 = 2(x-3)$$

$$\Rightarrow \quad y - 6 = 2x - 6$$

$$\Rightarrow \quad y = 2x \text{ or } 2x - y = 0$$ **Ans.**

Solution 3.

(b)

Marks (x)	No. of students (f)	$(f \cdot x)$	Cumulative frequency $(c.f.)$
0	1	0	1
1	3	3	4
2	6	12	10
3	10	30	20
4	5	20	25
5	5	25	30
	$\Sigma f = 30$	$\Sigma fx = 90$	

$$\therefore \quad \text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{90}{30} = 3$$

∴ Mean marks is 3.

Here, $n = 30$, which is even

$$\therefore \quad \text{Median} = \frac{\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n+1}{2}\right)^{th} \text{term}}{2}$$

$$= \frac{\left(\frac{30}{2}\right)^{th} \text{term} + \left(\frac{30}{2}+1\right)^{th} \text{term}}{2}$$

$$= \frac{15^{th}\text{term} + 16^{th}\text{term}}{2}$$

$$= \frac{3+3}{2}$$

∴ Median marks = 3

Since, the number 3 has maximum frequency 10.

∴ Mode = 3

∴ Mean = 3, Median = 3 and Mode = 3. **Ans.**

(c) Given, ∠SRT = 65° and SP is a tangent

∴ ∠TSR = 90°

(angle between the radius and tangent)

In Δ STR,

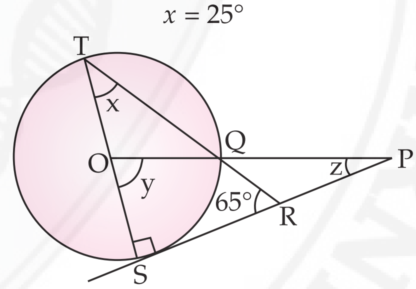
∠TSR + ∠SRT + ∠STR = 180°

(Angle sum property of triangle)

∴ ∠STR = 180° – (65° + 90°)

x = 180° – 155°

x = 25°



∠y = 2 ∠x

(Angle subtended at the centre is double that of the angle subtended by the arc at same centre)

∴ y = 2 × 25° = 50°

In Δ SPO,

∠SOP + ∠OSP + ∠SPO = 180°

(Angle sum property of triangle)

∴ ∠SPO = 180° – (90° + 50°)

z = 40°

Hence, x = 25°, y = 50° and z = 40° **Ans.**

Solution 4.

(a) Since, money deposited = ₹ 1,000 per month *i.e.,* P = ₹ 1,000

and number of months = 2 × 12 = 24 *i.e.,* n = 24 and r = 6%

(i) Interest earned in 2 years

$$= P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= 1,000 \times \frac{24(24+1)}{2 \times 12} \times \frac{6}{100}$$

= ₹ 1,500. **Ans.**

(ii) Maturity value = Sum deposited + Interest

= ₹ (1,000 × 24) + ₹ 1,500

= ₹ 25,500 **Ans.**

(b) Since $x = 3$ is the solution of the given equation $(K + 2)x^2 - Kx + 6 = 0$ …(i)

we get,

$(K + 2)(3)^2 - K(3) + 6 = 0$

$\Rightarrow 9K + 18 - 3K + 6 = 0$

$\Rightarrow 6K = -24$

$\Rightarrow K = -4$ **Ans.**

Now, putting K= – 4 in equation (i), we get

$\Rightarrow (-4 + 2)x^2 - (-4)x + 6 = 0$

$\Rightarrow -2x^2 + 4x + 6 = 0$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow x^2 - 3x + x - 3 = 0$

$\Rightarrow x(x - 3) + 1(x - 3) = 0$

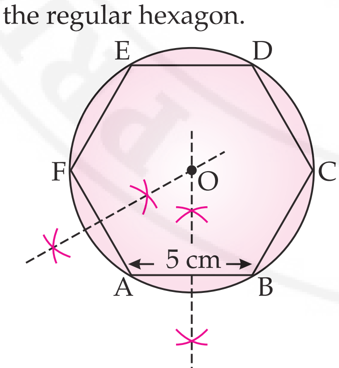
$\Rightarrow (x - 3)(x + 1) = 0$

$\Rightarrow x = 3$ or $x = -1$

∴ The other root is – 1. **Ans.**

(c) Steps of Construction :

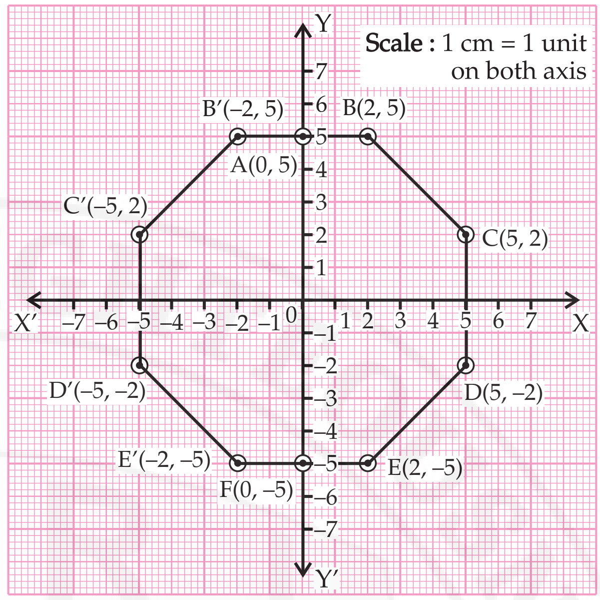
1. Construct a regular hexagon ABCDEF with each side 5 cm.
2. Draw the perpendicular bisectors of sides AB and AF which intersect each other at point O.
3. With O as centre and OA as radius, draw a circle which will pass through all the vertices of the regular hexagon.



SECTION—B

Solution 5.

(a) (i) Plot the given points on the graph as shown below :



(ii) and **(iii)**

Reflection of B (2, 5) on the Y-axis = (– 2 , 5) *i.e.*, B'

Reflection of C (5, 2) on the Y-axis = (– 5 , 2) *i.e.*, C'

Reflection of D (5, – 2) on the Y-axis = (– 5, – 2) *i.e.*, D'

Reflection of E (2, – 5) on the Y-axis = (– 2, – 5) *i.e.*, E'

(iv) Figure formed by BCDEE'D'C'B' is regular Octagon.

Solution 6.

(a) Given, a, b, c are in continued proportion.

$\therefore a : b = b : c$

$\Rightarrow \frac{a}{b} = \frac{b}{c}$

$\Rightarrow b^2 = ac$

L.H.S. $= (a + b + c)(a - b + c)$

$= a^2 - ab + ac + ab - b^2 + bc + ac - bc + c^2$

$= a^2 + 2ac - b^2 + c^2$

$= a^2 + 2b^2 - b^2 + c^2$ $(\because ac = b^2)$

$= a^2 + b^2 + c^2 =$ R.H.S. **Hence Proved.**

(b) (i) Coordinates of A are (4, 0)

(iii) Let the required ratio be K : 1 and the point Q be $(0, y)$

Here, $x_1 = 4, y_1 = 0, x_2 = -2, y_2 = -4$

We have, $x = \frac{Kx_2 + x_1}{K + 1}$

$\Rightarrow 0 = \frac{K(-2) + 4}{K + 1}$

$\Rightarrow 0 = -2K + 4$

$\Rightarrow 2K = 4$

$\Rightarrow K = 2$

$\Rightarrow K : 1 = 2 : 1$ **Ans.**

(iv) Equation of the line AC,

where, $x_1 = 4, y_1 = 0,$

$x_2 = -2, y_2 = -4$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - 0 = \frac{-4-0}{-2-4}(x-4)$$

$$\Rightarrow \quad y = \frac{2}{3}(x-4)$$

$$\Rightarrow \quad 3y = 2x - 8$$

$$\Rightarrow \quad 2x - 3y = 8$$ **Ans.**

(c)

Class Interval	Frequency (f)	Mean value (x)	fx
0 – 10	8	5	40
10 – 20	5	15	75
20 – 30	12	25	300
30 – 40	35	35	1225
40 – 50	24	45	1080
50 – 60	16	55	880
	$\Sigma f = 100$		$\Sigma fx = 3600$

$$\therefore \quad \text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{3600}{100} = 36$$

The mean of the given distribution is 36. **Ans.**

Solution 7.

(a) Volume of solid sphere of radius 2 cm

$$= \frac{4}{3}\pi(2)^3 = \frac{32}{3}\pi \text{ cm}^3$$

Volume of solid sphere of radius 4 cm

$$= \frac{4}{3}\pi(4)^3$$

$$= \frac{256}{3}\pi \text{ cm}^3$$

$$\text{Total Volume} = \left(\frac{32}{3}\pi + \frac{256}{3}\pi\right)\text{cm}^3$$

$$= 96\pi \text{ cm}^3$$

$\because$ Height of cone = 8 cm

$\therefore$ Volume of cone formed

$$= \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \quad 96\pi = \frac{1}{3}\pi r^2 \times 8$$

$$\Rightarrow \quad r^2 = \frac{96 \times 3}{8}$$

$$\Rightarrow \quad r = \sqrt{36}$$

$$\Rightarrow \quad r = 6 \text{ cm}$$

Hence, the radius of the cone formed is 6 cm.

Ans.

(b) Let $f(x) = ax^3 + 3x^2 - 9$ and $g(x) = 2x^3 + 4x + a$.

Since, the given polynomials leave the same remainder when divided by $x + 3$, so put $x + 3 = 0$ $\Rightarrow x = -3$ in $f(x)$ and $g(x)$.

By Remainder theorem,

$$f(-3) = g(-3)$$

$$\Rightarrow \quad a(-3)^3 + 3(-3)^2 - 9 = 2(-3)^3 + 4(-3) + a$$

$$\Rightarrow \quad -27a + 27 - 9 = -54 - 12 + a$$

$$\Rightarrow \quad -27a - a = -66 - 18$$

$$\Rightarrow \quad -28a = -84$$

$$\Rightarrow \quad a = 3.$$ **Ans.**

(c)

$$\text{L.H.S.} = \frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta}$$

$$= \frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}$$

$$\left(\because \tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}\right)$$

$$\Rightarrow \quad = \frac{\sin\theta}{\frac{\sin\theta - \cos\theta}{\sin\theta}} + \frac{\cos\theta}{\frac{\cos\theta - \sin\theta}{\cos\theta}}$$

$$= \frac{\sin^2\theta}{\sin\theta - \cos\theta} + \frac{\cos^2\theta}{\cos\theta - \sin\theta}$$

$$= \frac{\sin^2\theta}{\sin\theta - \cos\theta} - \frac{\cos^2\theta}{\sin\theta - \cos\theta}$$

$$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta - \cos\theta}$$

$$\left[\because a^2 - b^2 = (a-b)(a+b)\right]$$

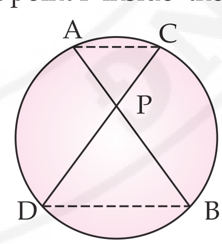
$$= \frac{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}{(\sin\theta - \cos\theta)}$$

$$= (\cos\theta + \sin\theta) = \text{R.H.S.}$$

Hence Proved.

Solution 8.

(a) Given : Chord AB and CD of a circle intersect each other at point P inside the circle.



To prove : AP × PB = CP × PD

Construction : Join AC and BD

Proof : In Δ APC and Δ BPD

$\angle$A = $\angle$D

(Angles of same segment)

$\angle$C = $\angle$B

(Angles of same segment)

$\therefore$ $\Delta APC \sim \Delta DPB$ (By AA axiom)

$\Rightarrow$ $\frac{AP}{PD} = \frac{CP}{PB}$

(corresponding sides of similar triangles)

$\Rightarrow$ $AP \times PB = CP \times PD$. **Hence Proved.**

(b) Given,

Number of white balls = 5

Number of red balls = 6

Number of green balls = 9

$\therefore$ Total number of outcomes = (5 + 6 + 9)

= 20

(i) P (getting a green ball) = $\frac{9}{20}$ **Ans.**

(ii) P (getting a white or red ball) = $\frac{5}{20} + \frac{6}{20}$

$= \frac{11}{20}$

$[\because P(A \cup B) = P(A)+P(B)]$

Ans.

(iii) P (getting neither a green ball nor a white ball) = P (getting a red ball) $\frac{6}{20} = \frac{3}{10}$ **Ans.**

(c) Given, sum invested = ₹ 9,600

N.V. of each share = ₹ 100

M.V. of each share = ₹ (100 + 20) = ₹ 120

rate of dividend = 8%

(i) Number of shares bought = $\frac{9,600}{120}$ = 80

Ans.

(ii) Selling price of one share = ₹ 160

Selling price of 80 shares = ₹ 80 × 160

= ₹ 12,800

Hence, Rohit's sales produced

= ₹ 12,800

Ans.

(iii) Market value of new share = ₹ 40

Investment = ₹ 12,800

New number of shares bought = $\frac{12,800}{40}$

= 320 **Ans.**

(iv) Dividend from original shares

= Number of shares × Rate of dividend × Face value of one share

= ₹ 80 × $\frac{8}{100}$ × 100

= ₹ 640

Annual dividend from new shares

= Number of shares × Rate of dividend × Face value of one share

= ₹ 320 × $\frac{10}{100}$ × 50

= ₹ 1,600

Change in two dividend

= ₹ (1,600 – 640)

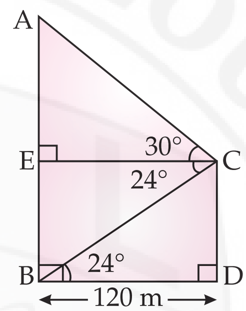
= ₹ 960

Dividend increase = ₹ 960. **Ans.**

Solution 9.

(a) Let, AB and CD be towers and BD = 120 m.

In right angled ΔBDC



$\tan 24° = \frac{CD}{BD}$

$0{\cdot}4452 = \frac{CD}{120}$

(using trigonometric table)

CD = 53·424 m

In right angled ΔAEC

$\tan 30° = \frac{AE}{EC} = \frac{AE}{BD}$ $(\because EC = BD)$

$\frac{1}{\sqrt{3}} = \frac{AE}{120}$

$AE = \frac{120}{\sqrt{3}}$

AE = 69·284 m

$\therefore$ AB = AE + EB $(\because EB = CD)$

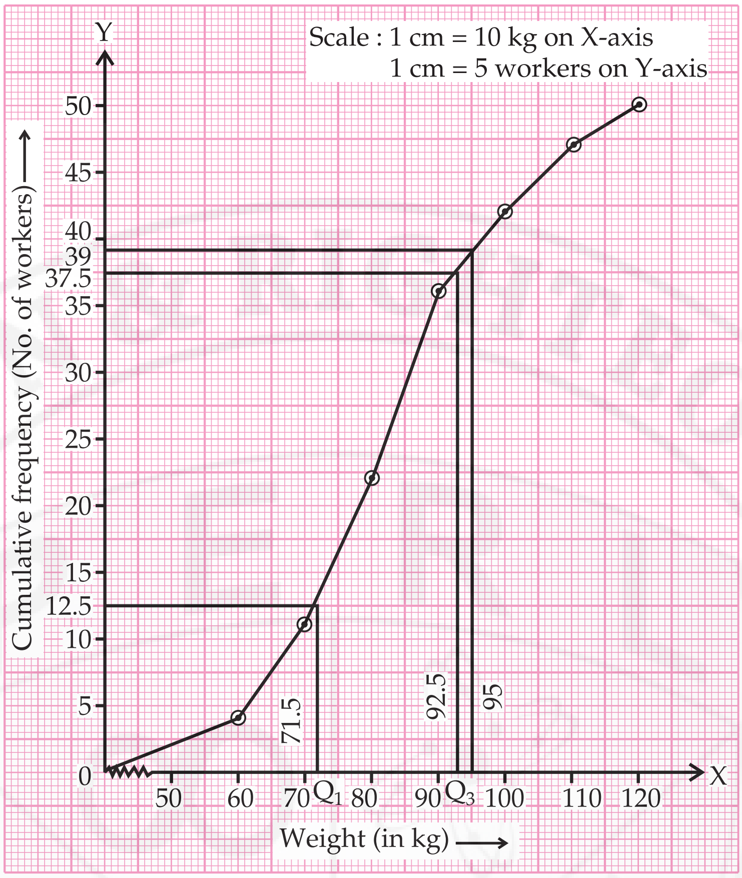
= 69·284 + 53·424

= 122·708 m

Hence, the height of the towers are 53·424 m and 122·708 m. **Ans.**

(b)

Weight (in kg)	No. of workers (*f*)	Cumulative frequency (*c.f.*)
50–60	4	4
60–70	7	11
70–80	11	22
80–90	14	36
90–100	6	42
100–110	5	47
110–120	3	50
	$N = \sum f = 50$	



Note : On Y-axis instead of 2 cm = 5 workers, we have taken 1 cm = 5 workers and on X-axis instead of 2 cm = 10 kg, we have taken 1 cm = 10 kg.

From graph,

(i) Upper quartile range

$$(Q_3) = \left(\frac{3N}{4}\right)^{th} \text{term}$$

$$= \left(\frac{3 \times 50}{4}\right)^{th} \text{term}$$

$$= 37{\cdot}5^{th} \text{ term} = 92{\cdot}5 \text{ kg}$$

Lower quartile range

$$(Q_1) = \left(\frac{N}{4}\right)^{th} \text{term}$$

$$= \left(\frac{50}{4}\right)^{th} \text{term}$$

$$= 12{\cdot}5^{th} \text{ term}$$

$$= 71{\cdot}5 \text{ kg.}$$ **Ans.**

(ii) From the graph, Number of workers who are under-weight *i.e.*, less than 95 are 39.

No. of workers who are over-weight are (50 – 39) = 11. **Ans.**

Solution 10.

(b) Given, $A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$

and $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$

$\therefore$ $AB = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 0+35 & 6+21 \\ 0+20 & 4+12 \end{bmatrix} = \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix}$$

and $5C = \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$

$\therefore$ $AB - 5C = \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$

$$= \begin{bmatrix} 35-5 & 27+25 \\ 20+20 & 16-30 \end{bmatrix}$$

$$= \begin{bmatrix} 30 & 52 \\ 40 & -14 \end{bmatrix}$$ **Ans.**

(c) **(i)** Given : Δ ABC, right angled at B and DE perpendicular to AC.

To prove : Δ ADE ~ Δ ACB.

In Δ ADE and Δ ACB,

$$\angle ABC = \angle AED = 90°$$

$$\angle A = \angle A \quad \text{(Common)}$$

$\therefore$ By AA axiom, Δ ADE ~ Δ ACB.

(ii) Given: AC = 13 cm, BC = 5 cm and AE = 4 cm

In right angled triangle ABC

$$AB^2 + BC^2 = AC^2$$

(by applying Pythagoras Theorem)

$$AB^2 + (5)^2 = (13)^2$$

$$AB = \sqrt{169-25} = 12 \text{ cm}$$

Since, the ΔADE and ΔACB are similar, then their corresponding sides will be proportional.

$$\therefore \quad \frac{AC}{AD} = \frac{AB}{AE} \Rightarrow \frac{13}{AD} = \frac{12}{4}$$

$$\Rightarrow \quad AD = \frac{13 \times 4}{12} = 4{\cdot}33 \text{ cm}$$

$$\text{and} \quad \frac{BC}{DE} = \frac{AB}{AE} \Rightarrow \frac{5}{DE} = \frac{12}{4}$$

$$DE = \frac{5 \times 4}{12} = 1{\cdot}67 \text{ cm.}$$ **Ans.**

(iii) In similar triangles, area of triangles are proportional to the square to the corresponding sides.

$$\frac{\text{Ar of }(\Delta\text{ ABC})}{\text{Ar of }(\Delta\text{ ADE})} = \frac{AB^2}{AE^2} = \frac{12^2}{4^2} = \frac{144}{16} = \frac{9}{1}$$

$$\Rightarrow \frac{\text{Ar of }(\Delta\text{ ADE}) + \text{Ar (quadrilateral BCED)}}{\text{Ar of }(\Delta\text{ ADE})} = 9$$

$$\Rightarrow \quad 1 + \frac{\text{Ar of (quadrilateral BCED)}}{\text{Ar of }(\Delta\text{ ADE})} = 9$$

$$\Rightarrow \quad \frac{\text{Ar of (quadrilateral BCED)}}{\text{Ar of }(\Delta\text{ ADE})} = 8$$

$$\Rightarrow \quad \frac{\text{Ar of }(\Delta\text{ ADE})}{\text{Ar of (quadrilateral BCED)}} = \frac{1}{8}$$ **Ans.**

Solution 11.

(a) Let, the two natural numbers be x and $8 - x$.

$$\text{Given,} \quad \frac{1}{x} - \frac{1}{8-x} = \frac{2}{15}$$

$$\Rightarrow \quad \frac{8-x-x}{x(8-x)} = \frac{2}{15}$$

$$(8 - 2x)\, 15 = 16x - 2x^2$$

$$120 - 30x = 16x - 2x^2$$

$$2x^2 - 46x + 120 = 0$$

$$x^2 - 23x + 60 = 0$$

On splitting the middle term, we get

$$x^2 - 20x - 3x + 60 = 0$$

$$x(x - 20) - 3(x - 20) = 0$$

$$(x - 3)(x - 20) = 0$$

$x = 3$ or $x = 20$ (neglect it)

(∵ Sum of two natural numbers is 8)

Thus, one number = 3 and other number = 8 – 3 = 5

∴ The natural numbers are 3 and 5. **Ans.**

(b) Given,

$$\frac{x^3+12x}{6x^2+8} = \frac{y^3+27y}{9y^2+27}$$

Applying componendo and dividendo, we get

$$\frac{x^3+12x+6x^2+8}{x^3+12x-6x^2-8} = \frac{y^3+27y+9y^2+27}{y^3+27y-9y^2-27}$$

$$[\because (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a-b)^3 = a^3 + 3ab^2 - 3a^2b - b^3]$$

$$\Rightarrow \quad \frac{(x+2)^3}{(x-2)^3} = \frac{(y+3)^3}{(y-3)^3}$$

Taking cube root on both the sides

$$\Rightarrow \quad \frac{x+2}{x-2} = \frac{y+3}{y-3}$$

Again using componendo and dividendo, we get

$$\frac{x+2+x-2}{x+2-x+2} = \frac{y+3+y-3}{y+3-y+3}$$

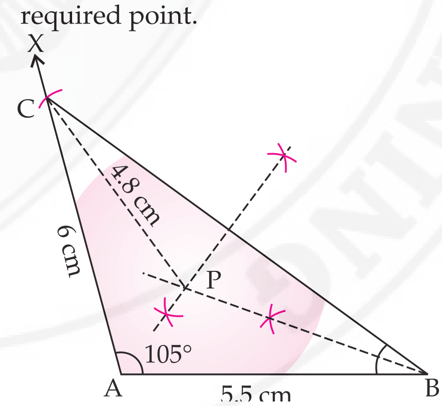
$$\frac{2x}{4} = \frac{2y}{6}$$

$$\frac{x}{y} = \frac{2}{3}$$

Hence, $x : y = 2 : 3$. **Ans.**

(c) Steps of construction:

1. Draw a line AB = 5·5 cm.
2. Now, from point A draw ∠XAB = 105° using compass.
3. Taking A as centre and 6 cm as radius draw arc on AX. Mark this point as C.
4. Join BC.
5. Draw bisector of ∠ABC and perpendicular bisector of BC, both intersecting at P. P is the required point.



Reason :

Since,

(i) P is on bisector of ∠ ABC, therefore, P is equidistant from BA and BC. **Ans.**

(ii) P is on perpendicular bisector of BC, therefore, P is equidistant from B and C. **Ans.**

(iii) Length of PC is 4·8 cm. **Ans.**

MATHEMATICS 2014

QUESTIONS

SECTION—A (40 Marks)

*(Attempt **all** questions from this Section)*

Question 1.

(a) *Ranbir borrows ₹ 20,000 at 12% per annum compound interest. If he repays ₹ 8,400 at the end of the first year and ₹ 9,680 at the end of the second year, find the amount of loan outstanding at the beginning of the third year.*** **[3]**

(b) *Find the values of x, which satisfy the inequation* $-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \le 2, x \in W$. *Show or represent the solution set on the number line, where W is the set of whole numbers.* **[3]**

(c) *A die has 6 faces marked by the given numbers as shown below :*

1	2	3	– 1	– 2	– 3

The die is thrown once. What is the probability of getting

(i) *a positive integer.*

(ii) *an integer greater than – 3.*

(iii) *the smallest integer.* **[4]**

Question 2.

(a) *Find x, y if* $\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2x \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2\begin{bmatrix} y \\ 3 \end{bmatrix}$. **[3]**

(b) *Shahrukh opened a Recurring Deposit Account in a bank and deposited ₹ 800 per month for* $1\frac{1}{2}$ *years. If he received ₹ 15,084 at the time of maturity, find the rate of interest per annum.* **[3]**

(c) *Calculate the ratio in which the line joining A (– 4, 2) and B (3, 6) is divided by point P (x, 3). Also, find* **(i)** *x*

(ii) *Length of AP.*** **[4]**

Question 3.

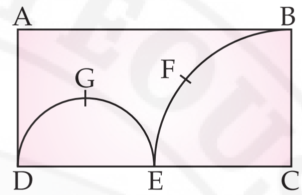
(a) *Without using trigonometric tables, evaluate.***

$\sin^2 34^\circ + \sin^2 56^\circ + 2 \tan 18^\circ \tan 72^\circ - \cot^2 30^\circ$ **[3]**

(b) *Using the Remainder and Factor Theorem, factorise the following polynomial :*

$x^3 + 10x^2 - 37x + 26.$ **[3]**

(c) *In the figure given below, ABCD is a rectangle. AB = 14 cm, BC = 7 cm. From the rectangle, a quarter circle BFEC and a semicircle DGE are removed. Calculate the area of the remaining piece of the rectangle.*** $\left(\text{Take } \pi = \frac{22}{7}\right)$ **[4]**



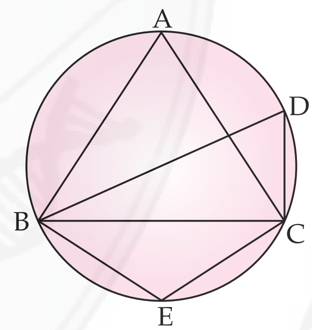
** Answer is not given due to change in the present syllabus.

Question 4.

(a) *The numbers 6, 8, 10, 12, 13, and x are arranged in an ascending order. If the mean of the observations is equal to the median, find the value of x.* **[3]**

(b) *In the given figure, ∠ DBC = 58°, BD is diameter of the circle. Calculate :*

(i) *∠ BDC* **(ii)** *∠ BEC* **(iii)** *∠BAC* **[3]**



(c) *Use graph paper to answer the following questions. (Take 2 cm = 1 unit on both axis).*

(i) *Plot the points A (– 4, 2) and B (2, 4).*

(ii) *A′ is the image of A when reflected in the Y-axis. Plot it on the graph paper and write the coordinates of A′.*

(iii) *B′ is the image of B when reflected in the line AA′. Write the coordinates of B′.*

(iv) *Write the geometric name of the figure ABA′B′.*

(v) *Name a line of symmetry of the figure formed.*** **[4]**

SECTION—B (40 Marks)

*(Attempt any **four** questions from this Section)*

Question 5.

(a) *A shopkeeper bought a washing machine at a discount of 20% from a wholesaler, the printed price of the washing machine being ₹ 18,000. The shopkeeper sells it to a consumer at a discount of 10% on the printed price. If the rate of sales tax is 8%, find :*

(i) *the VAT paid by the shopkeeper.*

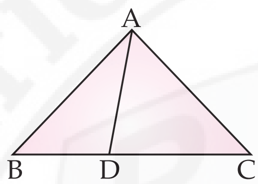
(ii) *the total amount that the consumer pays for the washing machine.*** **[3]**

(b) *If* $\frac{x^2+y^2}{x^2-y^2}=\frac{17}{8}$, *using the properties of proportion find the value of :*

(i) $x : y$.

(ii) $\frac{x^3+y^3}{x^3-y^3}$. **[3]**

(c) *In* ΔABC, $\angle ABC = \angle DAC$, *AB = 8cm, AC = 4 cm, AD = 5 cm.*



(i) *Prove that* $\Delta ACD \sim \Delta BCA$.

(ii) *Find the length of BC and CD.*

(iii) *Find area of* ΔACD : *area of* ΔABC. **[4]**

Question 6.

(a) *Find the value of 'a' for which the following points A (a, 3), B (2, 1) and C (5, a) are collinear. Hence, find the equation of the line.* **[3]**

(b) *Salman invests a sum of money in ₹ 50 shares, paying 15% dividend quoted at 20% premium. If his annual dividend is ₹ 600, calculate :*

(i) *the number of shares he bought.*

(ii) *his total investment.*

(iii) *the rate of return on his investment.* **[3]**

(c) *The surface area of a solid metallic sphere is 2464* cm^2. *It is melted and recast into solid right circular cones of radius 3.5 cm and height 7 cm. Calculate :*

(i) *the radius of the sphere.*

(ii) *the number of cones formed. (Take* $\pi = 22/7$*)* **[4]**

Question 7.

(a) *Calculate the mean of the distribution given below using the short cut method.*

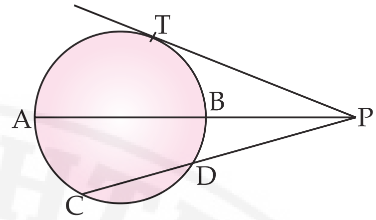
Marks	11–20	21–30	31–40	41–50	51–60	61–70	71–80
No. of students	2	6	10	12	9	7	4

[3]

(b) *In the figure given below, diameter AB and chord CD of a circle meet at P. PT is a tangent to the circle at T. CD = 7·8 cm, PD = 5 cm, PB = 4 cm. Find :*

(i) *AB.*

(ii) *The length of tangent PT.* **[3]**



(c) *Let* $A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$ *and* $C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$

Find $A^2 + AC - 5B$. **[4]**

Question 8.

(a) *The compound interest, calculated yearly, on a certain sum of money for the second year is ₹ 1320 and for the third year is ₹ 1452. Calculate the rate of interest and the original sum of money.*** **[3]**

(b) *Construct a* Δ *ABC with BC = 6·5 cm, AB = 5·5 cm, AC = 5 cm. Construct the incircle of the triangle. Measure and record the radius of the incircle.* **[3]**

(c) *The daily pocket expenses of 200 students in a school are given below : (Use a graph paper for this question.)*

Pocket expenses (in ₹)	Number of students (frequency)
0—5	10
5—10	14
10—15	28
15—20	42
20—25	50
25—30	30
30—35	14
35—40	12

Draw a histogram representing the above distribution and estimate the mode from the graph. **[4]**

Question 9.

(a) *If* $(x - 9) : (3x + 6)$ *is the duplicate ratio of 4 : 9, find the value of x using properties of proportion.* **[3]**

(b) *Solve for x using the quadratic formula. Write your answer correct to two significant figures.*

$$(x - 1)^2 - 3x + 4 = 0.$$ **[3]**

(c) *A page from the savings bank account of Priyanka is given below :*

Date	Particulars	Amount withdrawn (₹)	Amount deposited (₹)	Balance (₹)
3/4/2006	B/F			4000·00
5/4/2006	By Cash		2000·00	6000·00

** Answer is not given due to change in the present syllabus.

18/4/2006	By Cheque		6000·00	12000·00
25/5/2006	To Cheque	5000·00		7000·00
30/5/2006	By Cash		3000·00	10000·00
20/7/2006	By Self	4000·00		6000·00
10/9/2006	By Cash		2000·00	8000·00
19/9/2006	To Cheque	1000·00		7000·00

*If the interest earned by Priyanka for the period ending September, 2006 is ₹ 175, find the rate of interes.*** **[4]**

Question 10.

(a) *A two digit positive number is such that the product of its digits is 6. If 9 is added to the number, the digits interchange their places. Find the number.* **[4]**

(b) *The marks obtained by 100 students in a Mathematics test are given below :*

Marks	0—10	10—20	20—30	30—40	40—50	50—60	60—70	70—80	80—90	90—100
No. of Students	3	7	12	17	23	14	9	6	5	4

Draw an ogive for the given distribution on a graph sheet.

(Use a scale of 2 cm = 10 units on both axis).

Use the ogive to estimate the :

(i) *median.*

(ii) *lower quartile.*

(iii) *number of students who obtained more than 85% marks in the test.*

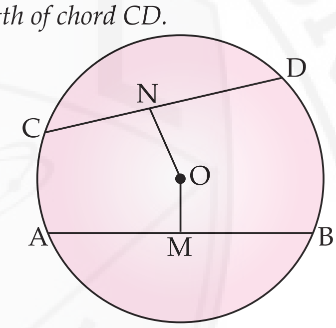
(iv) *number of students who did not pass in the test if the pass percentage was 35.* **[6]**

Question 11.

(a) *In the figure given below, O is the centre of the circle. AB and CD are two chords of the circle. OM is perpendicular to AB and ON is perpendicular to CD. AB = 24 cm, OM = 5 cm, ON = 12 cm. Find the :***

(i) *radius of the circle.*

(ii) *length of chord CD.* **[3]**



(b) *Prove the identity*

$(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \text{cosec}\,\theta$. **[3]**

(c) *An aeroplane at an altitude of 250 m observes the angle of depression of two boats on the opposite banks of a river to be 45° and 60° respectively. Find the width of the river. Write the answer correct to the nearest whole number.* **[4]**

Answers

SECTION—A

Solution 1.

(b) $-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2$

Taking, $-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3}$

$\Rightarrow \quad -\frac{17}{6} < \frac{1}{2} - \frac{2x}{3}$

$\Rightarrow \quad -\frac{17}{6} - \frac{1}{2} < -\frac{2x}{3}$

$\Rightarrow \quad \frac{-17-3}{6} < -\frac{2x}{3}$

$\Rightarrow \quad -\frac{20}{6} < -\frac{2x}{3} \Rightarrow \frac{10}{3} > \frac{2x}{3}$

$5 > x \Rightarrow x < 5$ ...(i)

Now, taking $\frac{1}{2} - \frac{2x}{3} \leq 2$

$\Rightarrow \quad -\frac{2x}{3} \leq 2 - \frac{1}{2}$

$\Rightarrow \quad -\frac{2x}{3} \leq \frac{3}{2}$

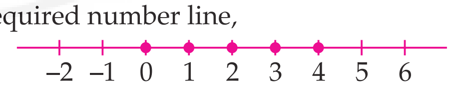
$\Rightarrow \quad -x \leq \frac{9}{4} \Rightarrow x \geq -\frac{9}{4}$ ...(ii)

From (i) and (ii), we get

$-\frac{9}{4} \leq x < 5 \Rightarrow -2\frac{1}{4} \leq x < 5$

But as $x \in$ W the solution set is, {0, 1, 2, 3, 4}.

Required number line,



Ans.

(c)

1	2	3	– 1	– 2	– 3

Total number of outcomes = 6,

$n(S) = 6$

** Answer is not given due to change in the present syllabus.

(i) A positive integer :

Favourable outcomes

$$n(P) = \{1, 2, 3\} = 3$$

$$\therefore \quad Q(P) = \frac{n(P)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$ **Ans.**

(ii) An integer greater than – 3 :

Favourable outcomes

$$n(g) = \{1, 2, 3, -1, -2\} = 5$$

$$\therefore \quad P(g) = \frac{n(g)}{n(S)} = \frac{5}{6}$$ **Ans.**

(iii) The smallest integer :

Favourable outcomes,

$$n(I) = \{-3\}$$

$$\therefore \quad P(I) = \frac{n(I)}{n(S)} = \frac{1}{6}$$ **Ans.**

Solution 2.

(a) Given, $\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2x \end{bmatrix} + 3\begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2\begin{bmatrix} y \\ 3 \end{bmatrix}$

$$\Rightarrow \quad \begin{bmatrix} 2+0 \\ -3+2x \end{bmatrix} + \begin{bmatrix} -6 \\ 3 \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2-6 \\ -3+2x+3 \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} -4 \\ 2x \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$$

On comparing, we get

$$\Rightarrow \quad 2y = -4, \; 2x = 6$$

$$\Rightarrow \quad y = -2, \quad x = 3$$

Thus required values are $x = 3, y = -2$. **Ans.**

(b) Given, P = ₹ 800 per month, $n = 1\frac{1}{2}$ years = 18 month.

Maturity value = ₹ 15,084

As we know,

$$I = \frac{Pn(n+1)r}{2 \times 12 \times 100}$$

$$M.V. = P \times n + I$$

$$\text{Maturity value} = P \times n + \frac{Pn(n+1)r}{2 \times 12 \times 100}$$

$$15{,}084 = 800 \times 18 + \frac{800 \times 18 \times 19 \times r}{2{,}400}$$

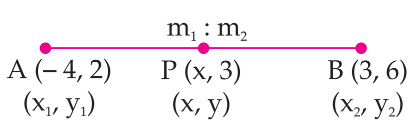
$$15{,}084 - 14{,}400 = 6 \times 19r$$

$$\frac{684}{6 \times 19} = r$$

$$\therefore \quad r = 6\% \text{ p.a.}$$ **Ans.**

(c) Let the ratio in which P divides AB be $m_1 : m_2$.

$\therefore$



y-coordinate of P

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$3 = \frac{6m_1 + 2m_2}{m_1 + m_2}$$

$$\Rightarrow \quad 3m_1 + 3m_2 = 6m_1 + 2m_2$$

$$\Rightarrow \quad 3m_1 - 6m_1 = 2m_2 - 3m_2$$

$$\Rightarrow \quad -3m_1 = -m_2$$

$$\Rightarrow \quad 3m_1 = m_2$$

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{1}{3}$$

$$\Rightarrow \quad m_1 : m_2 = 1 : 3$$ **Ans.**

(i) Now, $x = \dfrac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$

$$x = \frac{1 \times 3 + 3(-4)}{1+3}$$

$$= \frac{3 - 12}{4} = -\frac{9}{4}$$ **Ans.**

Solution 3.

(b) Given, $f(x) = x^3 + 10x^2 - 37x + 26$

$$f(1) = 1 + 10 - 37 + 26 = 37 - 37 = 0$$

Thus, $(x - 1)$ is a factor of $f(x)$.

$x - 1\overline{)\, x^3 + 10x^2 - 37x + 26\,}(\, x^2 + 11x - 26$

$x^3 - x^2$

$-\quad +$

$11x^2 - 37x$

$11x^2 - 11x$

$-\quad +$

$-26x + 26$

$-26x + 26$

$+\quad -$

0

Thus, $f(x) = (x - 1)(x^2 + 11x - 26)$

$$= (x - 1)(x^2 + 13x - 2x - 26)$$

$$= (x - 1)[x(x + 13) - 2(x + 13)]$$

$$= (x - 1)(x - 2)(x + 13)$$

Thus, required factors are $(x - 1)$, $(x - 2)$ and $(x + 13)$. **Ans.**

Solution 4.

(a) The numbers are 6, 8, 10, 12, 13 and x

$$n = 6$$

$$\text{Mean} = \frac{6 + 8 + 10 + 12 + 13 + x}{6}$$

$$\text{Mean} = \frac{49 + x}{6} \qquad \ldots(i)$$

For Median, $n = 6$ (even)

$$\therefore \quad \text{Median} = \frac{\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}}{2}$$

$$\text{Median} = \frac{3^{rd} \text{ term} + 4^{th} \text{term}}{2}$$

$$= \frac{10+12}{2}$$

$$= \frac{22}{2} = 11 \qquad \ldots\text{(ii)}$$

According to the question,

Median = Mean

$$\therefore \quad 11 = \frac{49+x}{6}$$

$$\Rightarrow \quad x = 66 - 49$$

$$\Rightarrow \quad x = 17.$$

The value of x is 17. **Ans.**

(b) Given, ∠ DBC = 58°, BD is the diameter.

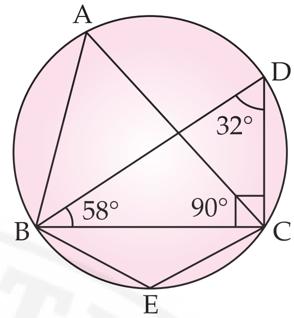
∴ ∠ BCD = 90°

(Angle in a semi-circle)

(i) Now, in Δ BDC

∠ BDC + 90° + 58° = 180°

(sum of the angles of a triangle)



∴ ∠ BDC = 180° – (90° + 58°)

= 32° **Ans.**

(ii) BECD is a cyclic quadrilateral.

∴ ∠ BEC + ∠ BDC = 180°

(Opp. angles of a cyclic quadrilateral)

∴ ∠ BEC = 180° – ∠ BDC

= 180° – 32° = 148° **Ans.**

(iii) ∠ BAC = ∠ BDC

= 32°

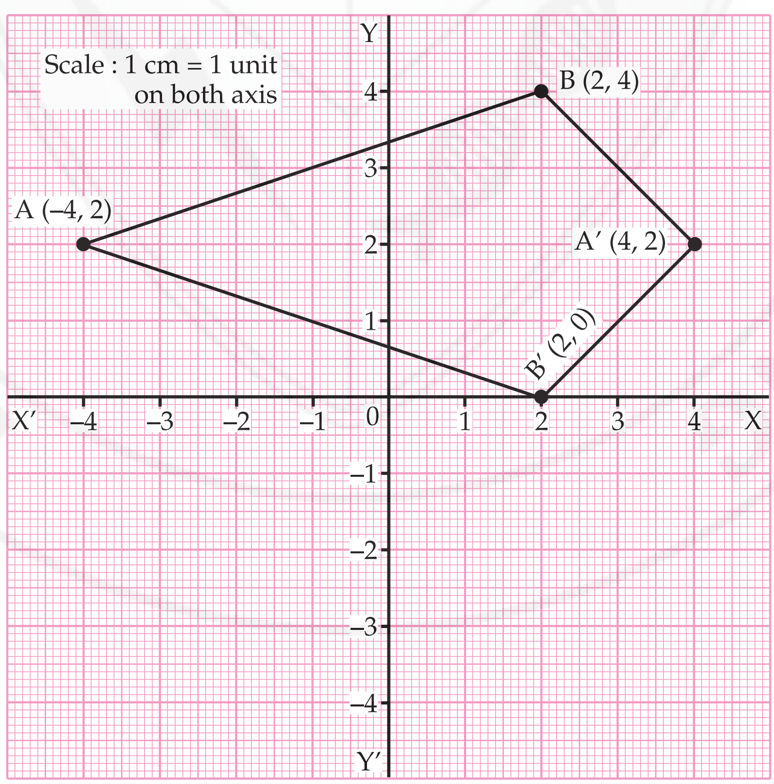
(Angles in the same segment of a circle are equal) **Ans.**

(c) **(i)** On the graph.

(ii) Coordinates of A′ = (4, 2).

(iii) Coordinates of B′ = (2, 0).

(iv) Geometric name of figure ABA′B′ is Kite. **Ans.**



Note : Instead of taking 2 cm = 1 unit on both axis, we have taken 1 cm = 1 unit on both the axis.

SECTION—B

Solution 5.

(b) (i)
$$\frac{x^2+y^2}{x^2-y^2} = \frac{17}{8}$$

Applying componendo and dividendo rule,

$$\frac{x^2+y^2+x^2-y^2}{x^2+y^2-x^2+y^2} = \frac{17+8}{17-8}$$

$$\Rightarrow \quad \frac{2x^2}{2y^2} = \frac{25}{9}$$

$$\Rightarrow \quad \frac{x^2}{y^2} = \frac{25}{9}$$

$$\Rightarrow \quad \frac{x}{y} = \frac{5}{3}$$

$$x : y = 5 : 3$$ **Ans.**

(ii)
$$\frac{x}{y} = \frac{5}{3}$$

Taking cube on both sides,

$$\frac{x^3}{y^3} = \frac{125}{27}$$

Applying componendo and dividendo rule,

$$\frac{x^3+y^3}{x^3-y^3} = \frac{125+27}{125-27}$$

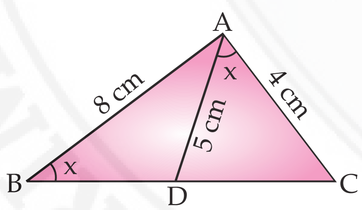
$$\frac{x^3+y^3}{x^3-y^3} = \frac{152}{98}.$$ **Ans.**

(c) Given, $\angle ABC = \angle DAC$

AB = 8 cm,

AC = 4 cm, AD = 5 cm.

(i) In Δ ACD and Δ BCA



$\angle ABC = \angle DAC$ (Given)

$\angle ACD = \angle BCA$ (Common)

$\Rightarrow$ Δ ACD ~ Δ BCA (By A-axiom).

Hence, Δ ACD ~ Δ BCA **Hence Proved.**

(ii) As we have,

Since Δ ACD ~ Δ BCA, then their corresponding sides will be proportional.

$$\frac{AC}{BC} = \frac{CD}{CA} = \frac{AD}{BA}$$

$$\Rightarrow \quad \frac{4}{BC} = \frac{CD}{4} = \frac{5}{8}$$

$$\Rightarrow \quad \frac{4}{BC} = \frac{5}{8}$$

$$\Rightarrow \quad BC = \frac{8\times 4}{5} = \frac{32}{5} = 6{\cdot}4 \text{ cm.}$$

and
$$\frac{CD}{4} = \frac{5}{8} \Rightarrow CD = \frac{5\times 4}{8}$$

$$\Rightarrow \quad CD = 2{\cdot}5 \text{ cm.}$$ **Ans.**

(iii) Area of similar triangles are proportional to the square of their corresponding sides.

$$\frac{\text{Area of } \Delta\, ACD}{\text{Area of } \Delta\, ABC} = \left(\frac{AC}{BC}\right)^2$$

$$= \left(\frac{4}{6.4}\right)^2 = \left(\frac{5}{8}\right)^2$$

Thus, area of Δ ACD : area of Δ ABC = 25 : 64.

Ans.

Solution 6.

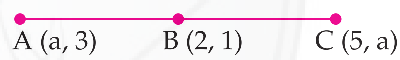
(a) Equation of line passing through AC is

Here, $x_2 = a$, $y_1 = 3$, $x_2 = 5$, $y_2 = a$

$$(y-y_1) = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$\Rightarrow \quad (y-3) = \left(\frac{a-3}{5-a}\right)(x-a)$$

As if A, B and C are collinear then B will satisfy it, *i.e.*,



$$(1-3) = \left(\frac{a-3}{5-a}\right)(2-a)$$

$$-2(5-a) = (a-3)(2-a)$$

$$-10+2a = 2a-6-a^2+3a$$

$$a^2-3a-4 = 0$$

$$a^2-4a+a-4 = 0$$

$$a(a-4)+1(a-4) = 0$$

$$(a-4)(a+1) = 0$$

$$\Rightarrow \quad a = 4 \text{ or } -1.$$ **Ans.**

Thus, required equation of straight line is

When, $a = 4$

$$(y-3) = \left(\frac{4-3}{5-4}\right)(x-4)$$

$$y-3 = \left(\frac{1}{1}\right)(x-4)$$

$$x-y-1 = 0$$

When, $a = -1$

$$(y-3) = \left(\frac{-1-3}{5+1}\right)(x+1)$$

$$(y-3) = \left(\frac{-4}{6}\right)(x+1)$$

$$y-3 = \frac{-2}{3}(x+1)$$

$$3y-9 = -2x-2$$

$$2x+3y-7 = 0$$ **Ans.**

(b) Face value = ₹ 50,

Dividend % = 15%

Market value = 50 + 20% of 50

= 50 + 10 = ₹ 60

Annual dividend = ₹ 600

(i) As we know,

Dividend % × (No. of shares × Face value) = Dividend

$\frac{15}{100} \times$ No. of shares $\times 50 = 600$

No. of shares $= \frac{600 \times 100}{15 \times 50} = 80$. **Ans.**

(ii) Total investment = 80 × Market value

$= 80 \times 60 =$ ₹ 4,800. **Ans.**

(iii) Rate of return on his investment

$= \left(\frac{\text{Total dividend}}{\text{Investment}} \times 100\right)\%$

$= \left(\frac{600}{4,800} \times 100\right)\%$

$= \left(\frac{100}{8}\right)\% = 12{\cdot}5\%$. **Ans.**

(c) (i) Surface area of sphere = 2464 cm^2

$4\pi r^2 = 2464$

$4 \times \frac{22}{7} \times r^2 = 2464$

$r^2 = \frac{2464 \times 7}{4 \times 22}$

$r^2 = 196$

$r = 14$ cm

Thus, radius of the sphere is 14 cm. **Ans.**

(ii) For cone, $r_1 = 3{\cdot}5$ cm, $h_1 = 7$ cm

Volume of cone $= \frac{1}{3}\pi r_1^2 h_1$

$= \frac{1}{3}\pi \times 3{\cdot}5 \times 3{\cdot}5 \times 7$

Volume of given sphere

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 14 \times 14 \times 14$

∴ Number of cones recast

$= \frac{\text{Volume of sphere}}{\text{Volume of cone}}$

$= \frac{\frac{4}{3}\pi \times 14 \times 14 \times 14}{\frac{1}{3}\pi \times 3.5 \times 3.5 \times 7}$

$= \frac{4 \times 14 \times 14 \times 14}{3.5 \times 3.5 \times 7}$

$= 4 \times 4 \times 4 \times 2 = 128$.

Thus, number of cones formed are 128. **Ans.**

Solution 7.

(a)

Class Interval (Inclusive form)	Class Interval (Exclusive form)	No. of Students (f_i)	x_i	$d_i = x - 45{\cdot}5$	$f_i d_i$
11—20	10·5–20·5	2	15·5	– 30	– 60 }
21—30	20·5–30·5	6	25·5	– 20	– 120 } – 280
31—40	30·5–40·5	10	35·5	– 10	– 100 }
41—50	40·5–50·5	12	45·5 = A	0	0
51—60	50·5–60·5	9	55·5	10	90 }
61—70	60·5–70·5	7	65·5	20	140 } 350
71—80	70·5–80·5	4	75·5	30	120 }
		$\Sigma f_i = 50$			$\Sigma f_i d_i = 70$

Assumed mean (A) $= 45{\cdot}5$

$\Sigma f_i = 50$, $\Sigma f_i d_i = 70$.

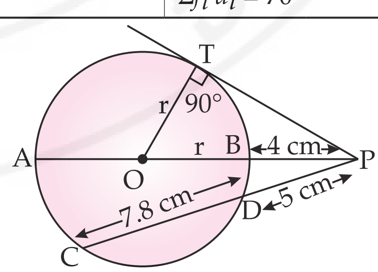
Mean $= A + \frac{\Sigma f_i d_i}{\Sigma f_i}$

$= 45{\cdot}5 + \frac{70}{50}$

$= 45{\cdot}5 + 1{\cdot}4$

$= 46{\cdot}9$. **Ans.**

(b) Given, CD = 7·8 cm, PD = 5 cm, PB = 4 cm and PT is a tangent.



As we know,

∵ Square of the length of tangent is equal to the product of the length of segments of the chord from the point of contact to point of intersection.

$$PT^2 = PD \times PC$$

$$\Rightarrow \quad PT^2 = PD \times (PD + CD)$$

$$= 5 \times (5 + 7.8)$$

$$\Rightarrow \quad PT^2 = 5 \times 12{\cdot}8$$

$$\Rightarrow \quad PT^2 = 64$$

$$\Rightarrow \quad PT = 8 \text{ cm}$$

Now in Δ POT

$$PO^2 = OT^2 + PT^2$$

(By Pythagoras Theorem)

$$\Rightarrow \quad (r + 4)^2 = r^2 + 64$$

$$\Rightarrow \quad r^2 + 16 + 8r = r^2 + 64$$

$$\Rightarrow \quad 8r = 48$$

$$\Rightarrow \quad r = 6$$

(i) Thus, $AB = 2r = 12$ cm

(ii) Length of tangent PT = 8 cm. **Ans.**

(c) $A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}, C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}.$

$$A^2 = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 2-2 \\ 0 & 0+4 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$5B = 5\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix}$$

$$AC = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -6-1 & 4+4 \\ 0+2 & 0-8 \end{bmatrix} = \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix}$$

$$\therefore \quad A^2 + AC - 5B = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix} - \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix}$$

$$\Rightarrow \quad = \begin{bmatrix} 4-7-20 & 0+8-5 \\ 0+2+15 & 4-8+10 \end{bmatrix}$$

$$\Rightarrow \quad A^2 + AC - 5B = \begin{bmatrix} -23 & 3 \\ 17 & 6 \end{bmatrix}.$$ **Ans.**

Solution 8.

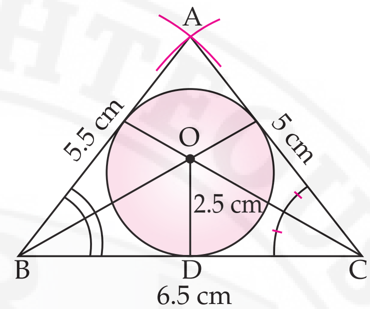
(b) Steps of construction :

1. Draw a line BC = 6·5 cm.
2. From point B and C draw an arc of 5·5 cm and 5 cm respectively and mark the intersecting point as A.
3. Join AB and AC.
4. Draw the bisectors of ∠ B and ∠ C. Let, the bisectors meet at point O.
5. From O, drop perpendicular on side BC. Let OD be the perpendicular drawn on BC.
6. With O as centre, and OD as radius. Draw a circle.

The circle so obtained is the required circle.

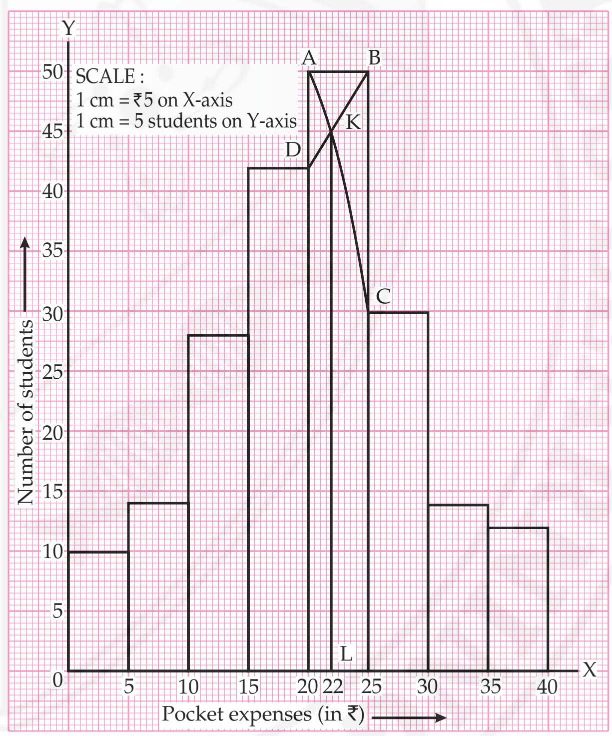
Given that,

BC = 6·5 cm, AB = 5·5 cm, AC = 5 cm



∴ Radius of incircle is 2·5 cm. **Ans.**

(c) Histogram on the graph paper.



Join A to C and B to D. AC and BD meet at K. Drop a perpendicular from K to X-axis at L.

∴ Mode = 22. **Ans.**

Solution 9.

(a) Given $(x - 9) : (3x + 6)$ is duplicate ratio of 4 : 9.

$$\therefore \quad \frac{x-9}{3x+6} = \left(\frac{4}{9}\right)^2$$

$$\Rightarrow \quad \frac{x-9}{3x+6} = \frac{16}{81}$$

$$\Rightarrow \quad 81(x - 9) = 16(3x + 6)$$

$$\Rightarrow \quad 81x - 729 = 48x + 96$$

$\Rightarrow \quad 81x - 48x = 96 + 729$

$\Rightarrow \quad 33x = 825$

$\Rightarrow \quad x = \frac{825}{33} = 25$

Thus, required value of x is 25. **Ans.**

(b) $(x-1)^2 - 3x + 4 = 0$

$\Rightarrow \quad x^2 + 1 - 2x - 3x + 4 = 0$

$\Rightarrow \quad x^2 - 5x + 5 = 0$

Since, middle term cannot be splitted, so we will compare it with $ax^2 + bx + c = 0$, we get

$a = 1, b = -5, c = 5$

By using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{25-20}}{2} = \frac{5 \pm \sqrt{5}}{2}$$

$$x = \frac{5 \pm 2.24}{2}$$

Taking +ve sign	Taking – ve sign
$x = \frac{5+2.24}{2}$	$x = \frac{5-2.25}{2}$
$= \frac{-7.24}{2}$	$= \frac{2.75}{2} = 1{\cdot}38$
$\Rightarrow x = 3{\cdot}62$	

Thus, required values are 3·62 and 1·38. **Ans.**

Solution 10.

(a) Let, the unit digit be x and tens digit will be $\frac{6}{x}$.

As two digit number is $(10a + b)$. Then, two digit number is $10 \times \frac{6}{x} + x = \frac{60}{x} + x$.

No. formed by interchanging digits $= \left(10x + \frac{6}{x}\right)$

From question,

$$\frac{60}{x} + x + 9 = 10x + \frac{6}{x}$$

$\Rightarrow \quad \frac{60 + x^2 + 9x}{x} = \frac{10x^2 + 6}{x}$

$\Rightarrow \quad 60 + x^2 + 9x = 10x^2 + 6$

$\Rightarrow \quad 9x^2 - 9x - 54 = 0$

$\Rightarrow \quad 9(x^2 - x - 6) = 0$

$\Rightarrow \quad x^2 - x - 6 = 0$

$\Rightarrow \quad x^2 - 3x + 2x - 6 = 0$

$\Rightarrow \quad x(x-3) + 2(x-3) = 0$

$\Rightarrow \quad (x-3)(x+2) = 0$

$\Rightarrow \quad x = -2 \text{ or } 3$

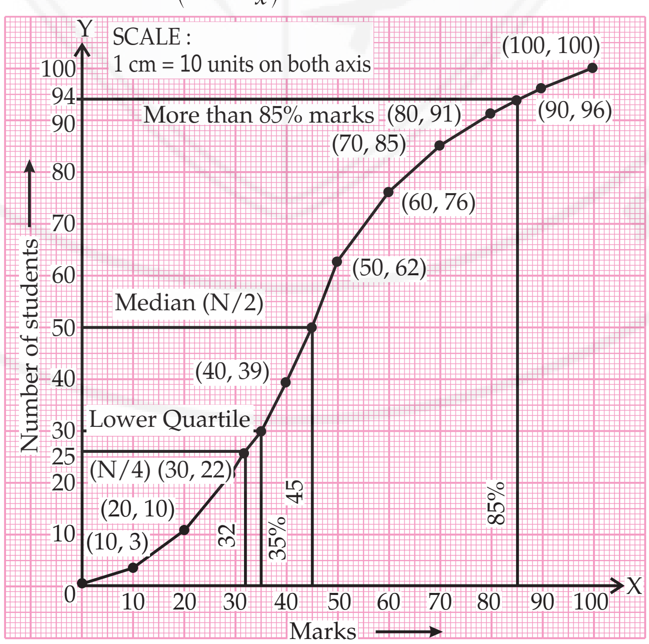
As x can't be negative.

So, required two digit number,

$$\frac{60}{x} + x = \frac{60}{3} + 3 = 23.$$ **Ans.**

(b)

Marks	c.f.	Points
Less than 10	3	(10, 3)
Less than 20	10	(20, 10)
Less than 30	22	(30, 22)
Less than 40	39	(40, 39)
Less than 50	62	(50, 62)
Less than 60	76	(60, 76)
Less than 70	85	(70, 85)
Less than 80	91	(80, 91)
Less than 90	96	(90, 96)
Less than 100	100	(100,100)



Note : Instead of 2 cm = 10 units, we have taken 1 cm = 10 units on both axis].

Using graph,

(i) Median $= \left(\frac{N}{2}\right)^{th}$ observation

$= \left(\frac{100}{2}\right)^{th}$ observation

$= 50^{th}$ observation

$= 45$ **Ans.**

(ii) Lower Quartile (Q_1)

$= \left(\frac{N}{4}\right)^{th}$ observation

$= \left(\frac{100}{4}\right)^{th}$ observation

$= 25^{th}$ observation $= 32$

Ans.

(iii) Number of students who obtained more than 85% marks

$= (100 - 94) = 6.$ **Ans.**

(iv) Number of students who did not pass if passing % of marks is 35

$= 30.$ **Ans.**

Solution 11.

(b) L.H.S. $= (\sin\theta + \cos\theta)(\tan\theta + \cot\theta)$

$= (\sin\theta + \cos\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$

$\left[\because \tan A = \frac{\sin A}{\cos A}, \cot A = \frac{\cos A}{\sin A}\right]$

$= (\sin\theta + \cos\theta)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)$

$= (\sin\theta + \cos\theta)\frac{1}{\sin\theta\cos\theta}$

$[\because \sin^2\theta + \cos^2\theta = 1]$

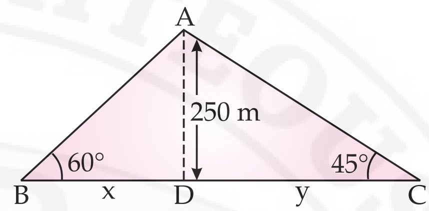
$= \frac{\sin\theta}{\sin\theta\cos\theta} + \frac{\cos\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\cos\theta} + \frac{1}{\sin\theta}$

$= \sec\theta + \text{cosec}\,\theta =$ R.H.S.

Hence Proved.

(c) Let aeroplane be at position A and BC be the river. Drop a perpendicular from A on BC let it intersect BC at D.



In ΔADB,

$$\tan 60° = \frac{AD}{BD}$$

$$\sqrt{3} = \frac{250}{x}$$

$$x = \frac{250}{\sqrt{3}} \text{ m} \quad \ldots(i)$$

In Δ ADC,

$$\tan 45° = \frac{AD}{DC}$$

$$1 = \frac{250}{y}$$

$$y = 250 \text{ m}$$

Thus, width of the river $= 250 + \frac{250}{\sqrt{3}} = 394$ m

Ans.

MATHEMATICS 2013

QUESTIONS

SECTION—A (40 Marks)

*(Attempt **all** questions from this Section)*

Question 1.

(a) Given $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

Find the matrix X such that $A + 2X = 2B + C$. **[3]**

(b) At what rate % p.a. will a sum of ₹ 4,000 yield ₹ 1,324 as compound interest in 3 years ?** **[3]**

(c) The median of the following observations 11, 12, 14, $(x - 2)$, $(x + 4)$, $(x + 9)$, 32, 38, 47 arranged in ascending order is 24. Find the value of x and hence find the mean. **[4]**

Question 2.

(a) What number must be added to each of the numbers 6, 15, 20 and 43 to make them proportional ? **[3]**

(b) If $(x - 2)$ is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by $(x - 3)$, it leaves a remainder 52, find the values of a and b. **[3]**

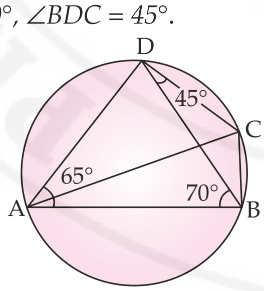
(c) Draw a histogram for the following frequency distribution and find the mode from the graph : **[4]**

Class	0–5	5–10	10–15	15–20	20–25	25–30
Frequency	2	5	18	14	8	5

Question 3.

(a) Without using tables evaluate 3 cos 80°. cosec 10° + 2 sin 59° sec 31°.** **[3]**

(b) In the given figure, ⇒ $\angle BAD = 65°$ $\angle ABD = 70°$, $\angle BDC = 45°$.



(i) Prove that AC is a diameter of the circle.

(ii) Find $\angle ACB$ **[3]**

(c) AB is a diameter of a circle with centre C = (– 2, 5). If A = (3, – 7). Find

(i) the length of radius AC**

(ii) the coordinates of B. **[4]**

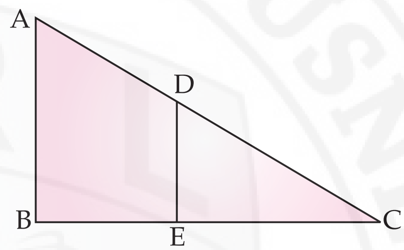
** Answer is not given due to change in the present syllabus.

Question 4.

(a) Solve the following equation and calculate the answer correct to two decimal places :

$$x^2 - 5x - 10 = 0.$$ **[3]**

(b) In the given figure, AB and DE are perpendicular to BC.



(i) Prove that $\Delta ABC \sim \Delta DEC$

(ii) If AB = 6 cm; DE = 4 cm and AC = 15 cm. Calculate CD.

(iii) Find the ratio of the area of ΔABC : area of ΔDEC. **[3]**

(c) Using a graph paper, plot the points A (6, 4) and B (0, 4).

(i) Reflect A and B in the origin to get the images A′ and B′.

(ii) Write the coordinates of A′ and B′.

(iii) State the geometrical name for the figure ABA′B′.

(iv) Find its perimeter. **[4]**

SECTION—B (40 Marks)

*(Attempt any **four** questions from this Section)*

Question 5.

(a) Solve the following inequation, write the solution set and represent it on the number line :

$$-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}, x \in R,$$

Where R is a set of real numbers. **[3]**

(b) Mr. Britto deposits a certain sum of money each month in a Recurring Deposit Account of a bank. If the rate of interest is of 8% per annum and Mr. Britto gets ₹ 8,088 from the bank after 3 years, find the value of his monthly instalment. **[3]**

(c) Salman buys 50 shares of face value ₹ 100 available at ₹ 132.

(i) What is his investment ?

(ii) *If the dividend is 7·5%, what will be his annual income ?*

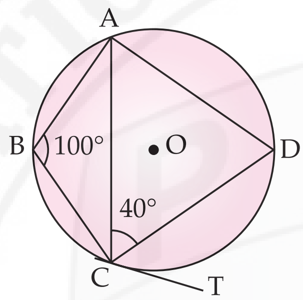
(iii) *If he wants to increase his annual income by ₹ 150, how many extra shares should he buy ?* [4]

Question 6.

(a) *Show that,*

$$\sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{\sin A}{1+\cos A}$$ [3]

(b) *In the given circle with centre O, ∠ ABC = 100°, ∠ACD = 40° and CT is a tangent to the circle at C. Find ∠ ADC and ∠ DCT.* [3]



(c) *Given below are the entries in a Savings Bank A/c pass book :*** [4]

Date	Particulars	With-drawals	Deposit	Balance
Feb. 8	B/F	—	—	₹ 8,500
Feb. 18	To Self	₹ 4,000	—	—
April 12	By Cash	—	₹ 2,230	—
June 15	To Self	₹ 5,000	—	—
July 8	By Cash	—	₹ 6,000	—

Calculate the interest for six months from February to July at 6% p.a.

Question 7.

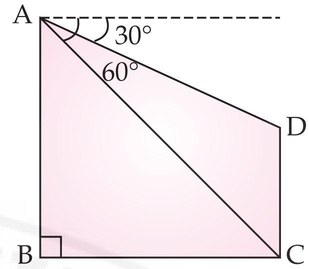
(a) *In ΔABC, A (3, 5), B (7, 8) and C (1, – 10). Find the equation of the median through A.* [3]

(b) *A shopkeeper sells an article at the listed price of ₹ 1,500 and the rate of VAT is 12% at each stage of sale. If the shopkeeper pays a VAT of ₹ 36 to the Government, what was the price, inclusive of Tax, at which the shopkeeper purchased the article from the wholesaler ?*** [3]

(c) *In the figure given, from the top of a building AB = 60 m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. Find :*

(i) *the horizontal distance between AB and CD.*

(ii) *the height of the lamp post.* [4]



Question 8.

(a) *Find x and y if* $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$. [3]

(b) *A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2·5 cm and height 8 cm. Calculate the number of cones formed.* [3]

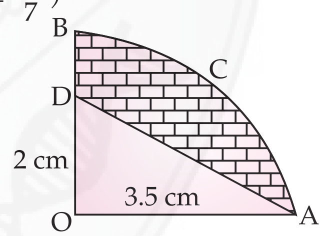
(c) *Without solving the following quadratic equation, find the value of 'p' for which the given equation has real and equal roots :*

$$x^2 + (p-3)x + p = 0$$ [4]

Question 9.

(a) *In the figure alongside, OAB is a quadrant of a circle. The radius OA = 3·5 cm and OD = 2 cm. Calculate the area of the shaded portion.***

(Take $\pi = \frac{22}{7}$*)* [3]



(b) *A box contains some black balls and 30 white balls. If the probability of drawing a black ball is two-fifths of a white ball, find the number of black balls in the box.* [3]

(c) *Find the mean of the following distribution by step deviation method :* [4]

Class interval	20–30	30–40	40–50	50–60	60–70	70–80
Frequency	10	6	8	12	5	9

Question 10.

(a) *Using a ruler and compasses only :*

(i) *Construct a triangle ABC with the following data : AB = 3·5 cm, BC = 6 cm and ∠ ABC = 120°.*

(ii) *In the same diagram, draw a circle with BC as diameter. Find a point P on the circumference of the circle which is equidistant from AB and BC.*

(iii) *Measure ∠ BCP.* [4]

(b) *The marks obtained by 120 students in a test are given below :*

** Answer is not given due to change in the present syllabus.

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
No. of Students	5	9	16	22	26	18	11	6	4	3

Draw an ogive for the given distribution on a graph sheet.

Use suitable scale for ogive to estimate the following :

(i) *The median.*

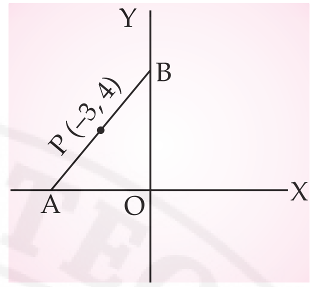
(ii) *The number of students who obtained more than 75% marks in the test.*

(iii) *The number of students who did not pass the test if minimum marks required to pass is 40.*

[6]

Question 11.

(a) *In the figure given below, the line segment AB meets X-axis at A and Y-axis at B. The point P (– 3, 4) on AB divides it in the ratio 2 : 3. Find the coordinates of A and B.* **[3]**



(b) *Using the properties of proportion, solve for x, given*

$$\frac{x^4+1}{2x^2} = \frac{17}{8}.$$ **[3]**

(c) *A shopkeeper purchase a certain number of books for ₹ 960. If the cost per book was ₹ 8 less, the number of books that could be purchased for ₹ 960 would be 4 more. Write an equation, taking the original cost of each book to be ₹ x, and solve it to find the original cost of the books.* **[4]**

Answers

SECTION—A

Solution 1.

(a) Given, $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} = B \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

$$A + 2X = 2B + C$$

$$\begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = 2\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$2X = \begin{bmatrix} -6 & 4 \\ 8 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$$

$$2X = \begin{bmatrix} -6+4-2 & 4+0+6 \\ 8+0-2 & 0+2-0 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$

$$\therefore \quad X = \begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix}$$ **Ans.**

(c) 11, 12, 14, $(x - 2)$, $(x + 4)$, $(x + 9)$, 32, 38, 47

Since, $n = 9$, odd

$\therefore$ Median $= \left(\frac{9+1}{2}\right)^{th}$ observation

$= 5^{th}$ observation $= (x + 4)$

Median is 24 for the given data

$\therefore \quad 24 = x + 4$

$24 - 4 = x$

$\Rightarrow \quad x = 20$ **Ans.**

$\therefore$ Observation are 11, 12, 14, (20 – 2), (20 + 4), (20 + 9), 32, 38, 47

or 11, 12, 14, 18, 24, 29, 32, 38, 47

$$\text{Mean} = \overline{X} = \frac{11 + 12 + 14 + 18 + 24 + 29 + 32 + 38 + 47}{9}$$

$$= \frac{225}{9} = 25$$

The mean of the given data is 25. **Ans.**

Solution 2.

(a) Let the number to be added to each of the numbers 6, 15, 20 and 43 be x.

$\therefore$ $6 + x$, $15 + x$, $20 + x$ and $43 + x$ are in proportion

$\Rightarrow \quad 6 + x : 15 + x :: 20 + x : 43 + x$

$\Rightarrow \quad \frac{6+x}{15+x} = \frac{20+x}{43+x}$

$\Rightarrow \quad (6 + x)(43 + x) = (15 + x)(20 + x)$

$\Rightarrow \quad 258 + 6x + 43x + x^2 = 300 + 15x + 20x + x^2$

$\Rightarrow \quad 258 + 49x + x^2 = 300 + 35x + x^2$

$\Rightarrow \quad 49x - 35x + x^2 - x^2 = 300 - 258$

$\Rightarrow \quad 49x - 35x = 300 - 258$

$\Rightarrow \quad 14x = 42$

$\Rightarrow \quad x = \frac{42}{14} = 3$

$\therefore$ Required number is 3. **Ans.**

(b) Let, $f(x) = 2x^3 + ax^2 + bx - 14$ …(i)

As $(x - 2)$ is a factor of equation (i)

$\therefore$ Putting $x - 2 = 0$

$\Rightarrow$ $x = 2$ in equation (i)

We get, $f(2) = 0$

and $f(2) = 2(2)^3 + a(2)^2 + b(2) - 14$

$0 = 16 + 4a + 2b - 14$

$\therefore$ $4a + 2b = -2$

or $2a + b = -1$ ...(ii)

Again, when $f(x)$ is divided by $(x - 3)$, it leaves remainder 52.

Putting $x - 3 = 0$

$\Rightarrow$ $x = 3$

We get, $f(3) = 52$

$\Rightarrow$ $f(3) = 2(3)^3 + a(3)^2 + b(3) - 14$

and, $52 = 54 + 9a + 3b - 14$

$\therefore$ $52 = 9a + 3b + 40$

$\Rightarrow$ $52 - 40 = 9a + 3b$

$\Rightarrow$ $12 = 9a + 3b$

or $4 = 3a + b$ ...(iii)

Solving (ii) and (iii)

$$\begin{array}{r} 3a + b = 4 \\ 2a + b = -1 \\ - \quad - \quad + \\ \hline a = 5 \end{array}$$

Ans.

Substitute $a = 5$ in equation (iii),

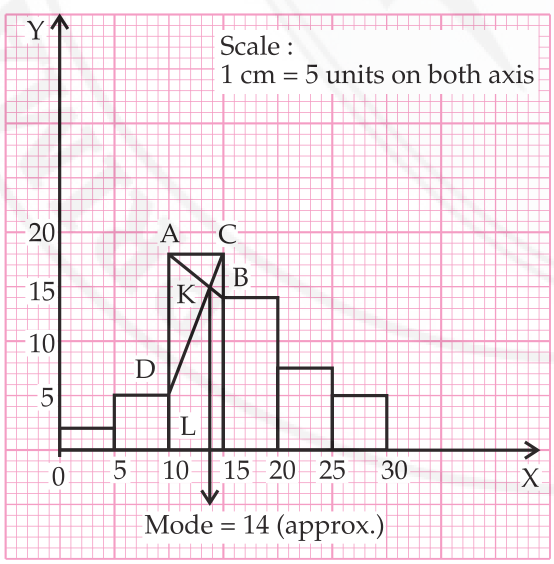
$\Rightarrow$ $3 \times 5 + b = 4$

$\Rightarrow$ $15 + b = 4$

$\Rightarrow$ $b = 4 - 15$

$\Rightarrow$ $b = -11$ **Ans.**

(c)



Using graph, in the biggest bar of class interval 10-15, we will join A to B and D to C. AB and CD meet at K. From K, drop a perpendicular on X-axis at L. Therefore, mode is 14. **Ans.**

Solution 3.

(b) Given : $\angle BAD = 65°$

$\angle ABD = 70°$

$\angle BDC = 45°$

(i) In ΔABD,

$\angle BAD + \angle ABD + \angle ADB = 180°$

(Sum of three angles of a Δ)

$65° + 70° + \angle ADB = 180°$

$\therefore$ $\angle ADB = 180° - (65° + 70°) = 45°$

$\because$ $\angle ADC = \angle ADB + \angle BDC$

$\Rightarrow$ $= 45° + 45° = 90°$

$\Rightarrow$ AC is the diameter of the circle.

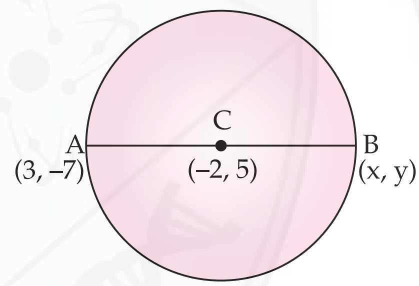
[Angle in a semi-circle is 90°] **Hence Proved.**

(ii) $\angle ACB = \angle ADB = 45°$

(Angles in the same segment of a circle)

Ans.

(c) **(ii)** As 'C' is mid-point of AB



$$-2 = \frac{3 + x}{2} \text{ and } 5 = \frac{-7 + y}{2}$$

[By mid-point formula]

or $-4 = 3 + x$ and $10 = -7 + y$

$x = -7$ and $y = 17$

$\therefore$ Coordinates of B are $(-7, 17)$. **Ans.**

Solution 4.

(a) Given equation is, $x^2 - 5x - 10 = 0$

We know,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(As, $a = 1, b = -5$ and $c = -10$)

$$\Rightarrow \quad x = \frac{5 \pm \sqrt{25 - 4 \times 1 \times (-10)}}{2}$$

$$= \frac{5 \pm \sqrt{65}}{2}$$

$$= \frac{5 \pm 8.062}{2}$$

$$x_1 = \frac{5 + 8.062}{2}$$

$$= \frac{13.062}{2}$$

$= 6{\cdot}531$

$= 6{\cdot}53$

and, $x_2 = \frac{5 - 8{\cdot}062}{2}$

$= \frac{-3{\cdot}062}{2} = 1{\cdot}531$

$= -1{\cdot}53$ **Ans.**

(b) **(i)** Given, in ΔABC,

$AB \perp BC$

$DE \perp BC$

To prove : $\Delta ABC \sim \Delta DEC$

Proof : In ΔABC and ΔDEC

$\angle ABC = \angle DEC = 90°$ (Given)

$\angle C = \angle C$ (Common)

$\therefore$ $\Delta ABC \sim \Delta DEC$ (By AA criteria)

Hence Proved.

(ii) $AB = 6$ cm, $DE = 4$ cm

$AC = 15$ cm, $CD = ?$

Since $\Delta ABC \sim \Delta DEC$

$\therefore \frac{AB}{DE} = \frac{AC}{CD}$

(Corresponding sides of similar triangles are proportional)

$\Rightarrow \frac{6}{4} = \frac{15}{CD}$

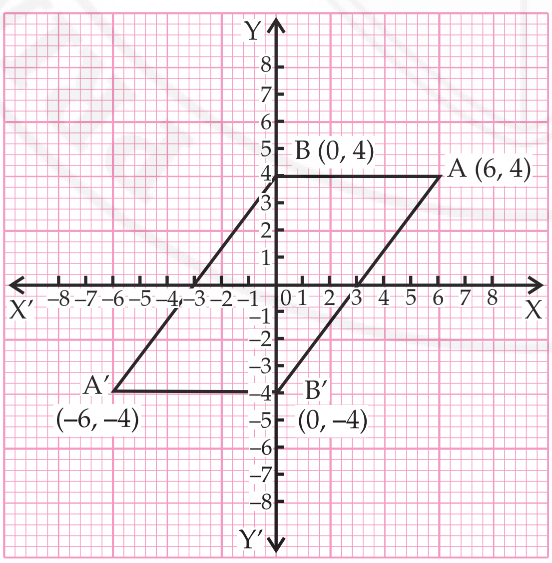
$\Rightarrow CD = \frac{15 \times 4}{6} = 10$ cm. **Ans.**

(iii) $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEC} = \frac{AB^2}{DE^2}$ (Area theorem)

$= \frac{36}{16}$

$= \frac{9}{4}$ or 9 : 4 **Ans.**

(c) **(i)**



(ii) Coordinates of A' (– 6, – 4) and B' (0, – 4). **Ans.**

(iii) ABA'B' is a parallelogram. **Ans.**

(iv) From the figure, AB = 6, BB' = 8, A'B' = 6.

In right angled $\Delta ABB'$,

$(AB')^2 = (AB)^2 + (BB')^2$

$= 6^2 + 8^2 = 100$

$\therefore$ $AB' = 10 = A'B$

(ABA'B' is a parallelogram)

$\therefore$ Perimeter of ABA'B'

$= AB + BA' + A'B' + AB'$

$= 6 + 10 + 6 + 10$

$= 32$ units **Ans.**

SECTION—B

Solution 5.

(a) $-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}, x \in R$

$-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3}$

$-\frac{x}{3} \le \frac{x}{2} - \frac{4}{3}$

$\frac{4}{3} \le \frac{x}{2} + \frac{x}{3}$

$\frac{4}{3} \le \frac{5x}{6}$

$\frac{6}{5} \times \frac{4}{3} \le x$

$\frac{8}{5} \le x$ ...(i)

$\frac{x}{2} - \frac{4}{3} < \frac{1}{6}$

$\frac{x}{2} < \frac{1}{6} + \frac{4}{3}$

$\frac{x}{2} < \frac{1+8}{6}$

$x < \frac{9 \times 2}{6}$

$x < 3$ ...(ii)

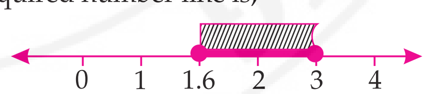
From equations (i) and (ii), we get

$\frac{8}{5} \le x < 3$

or $1{\cdot}6 \le x < 3$

$\therefore$ Solution set $\{x : 1{\cdot}6 \le x < 3, x \in R\}$

Required number line is,



Ans.

(b) Let the monthly instalment be ₹ x.

Here, $n = 36$, M.V. = ₹ 8,088, $r = 8\%$ p.a.,

$\therefore$ $I = P\frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$

$M.V. = P \times n + I$

$8{,}088 = x \times 36 + \left[\frac{x \times 36 \times 37}{2 \times 12} \times \frac{8}{100}\right]$

$8{,}088 = 36x + \frac{111x}{25}$

$$8{,}088 = \frac{900x + 111x}{25}$$

$$8{,}088 \times 25 = 1011x$$

$$\therefore \quad x = \frac{8088 \times 25}{1011} = 200.$$

$\therefore$ Monthly instalment is ₹ 200. **Ans.**

(c) Given, Number of shares = 50

F.V. = ₹ 100

M.V. = ₹ 132

(i) Investment = M.V. × Number of shares

= 132 × 50 = ₹ 6,600 **Ans.**

(ii) Dividend on 1 share

= 7·5% of 100

$= \frac{7 \cdot 5 \times 100}{100} =$ ₹ 7·5

$\therefore$ Annual Income (A.I.)

= Dividend on 1 share × No. of shares

= 7·5 × 50

= ₹ 375 **Ans.**

(iii) New annual income required

= ₹ (375 + 150) = ₹ 525

$\therefore$ New number of shares $= \frac{525}{7 \cdot 5} = 70$

$\therefore$ No. of extra share he should buy

= 70 – 50 = 20. **Ans.**

Solution 6.

(a) To prove, $\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{\sin A}{1 + \cos A}$

$$\text{L.H.S.} = \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \sqrt{\frac{1 - \cos A}{1 + \cos A} \times \frac{1 + \cos A}{1 + \cos A}}$$

$$= \sqrt{\frac{(1 - \cos^2 A)}{(1 + \cos A)^2}}$$

$$= \sqrt{\frac{\sin^2 A}{(1 + \cos A)^2}}$$

$$= \sqrt{\left(\frac{\sin A}{1 + \cos A}\right)^2}$$

$$= \frac{\sin A}{1 + \cos A} = \text{R.H.S.}$$

Hence Proved.

(b) Given, ∠ ABC = 100°, ∠ ACD = 40°

and CT is a tangent at C.

∠ ABC + ∠ ADC = 180°

(Opposite angles of a cyclic quadrilateral)

100° + ∠ ADC = 180°

$\therefore$ ∠ ADC = 180° – 100° = 80°. **Ans.**

Now, in Δ ACD

∠ ACD + ∠ ADC + ∠ CAD = 180°

(sum of angles of a Δ)

40° + 80° + ∠ CAD = 180°

∠ CAD = 180° – 120° = 60°

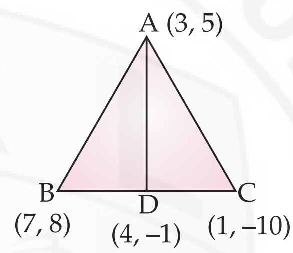
Now, ∠ DCT = ∠ CAD = 60°

(Alternate segment theorem) **Ans.**

Solution 7.

(a) Given, A (3, 5), B (7, 8) and C (1, – 10) are 3 co-ordinates of Δ. Median is drawn from A at BC.

Coordinates of D $\equiv \left(\frac{7+1}{2}, \frac{8-10}{2}\right) \equiv (4, -1)$



(Mid point formula)

Now, equation of AD {Median through A}

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$x_1 = 3, \quad x_2 = 4$$

$$y_1 = 5, \quad y_2 = -1$$

$$y - 5 = \frac{-1 - 5}{4 - 3}(x - 3)$$

$$y - 5 = -6(x - 3)$$

$$y - 5 = -6x + 18$$

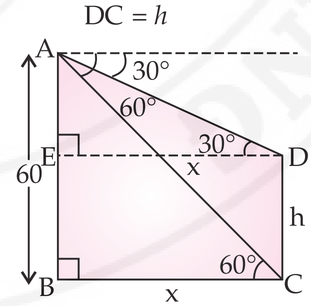
or $6x + y - 23 = 0$ **Ans.**

(c) We draw DE ⊥ AB

Let BC = x = ED

AB = 60 (given)

DC = h



$\Rightarrow$ BE = CD = h

$\therefore$ AE = AB – BE

= 60 – h

(i) In Δ ABC, $\frac{AB}{BC} = \tan 60°$

$$\frac{60}{x} = \sqrt{3}$$

$\Rightarrow \quad x = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{60\sqrt{3}}{3}$

$= 20\sqrt{3}$ m

Horizontal distance between lamp post and building is $20\sqrt{3}$ m. **Ans.**

(ii) In Δ AED, $\frac{AE}{ED} = \tan 30°$

$$\frac{60-h}{x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{60-h}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$\Rightarrow \quad 60 - h = 20$

$\Rightarrow \quad h = 60 - 20 = 40$ m.

∴ Height of lamp post is 40 m. **Ans.**

Solution 8.

(a) Given, $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x+3x \\ 2y+4y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix} \Rightarrow \begin{bmatrix} 5x \\ 6y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$

On comparing, we get

$\Rightarrow \quad 5x = 5$

$\Rightarrow \quad x = 1$ **Ans.**

and $\quad 6y = 12$

$\Rightarrow \quad y = 2$ **Ans.**

(b) Given, in sphere, $r = 15$ cm and in cone, $r = 2{\cdot}5$ cm, $h = 8$ cm.

$$\text{Number of cones} = \frac{\text{Volume of solid sphere}}{\text{Volume of 1 cone}}$$

$$= \frac{\frac{4}{3}\pi(15)^3}{\frac{1}{3}\pi(2{\cdot}5)^2 \times 8}$$

$$= \frac{4 \times 15 \times 15 \times 15}{2{\cdot}5 \times 2{\cdot}5 \times 8} = 270$$

∴ Number of cones formed are 270. **Ans.**

(c) Given quadratic equation is,

$$x^2 + (p-3)x + p = 0$$

Here $a = 1, b = p - 3, c = p$

For real and equal roots

$D = b^2 - 4ac = 0$

$\Rightarrow (p-3)^2 - 4 \times 1 \times p = 0$

$\Rightarrow p^2 - 6p + 9 - 4p = 0$

$\Rightarrow p^2 - 10p + 9 = 0$

$\Rightarrow p^2 - p - 9p + 9 = 0$

$\Rightarrow p(p-1) - 9(p-1) = 0$

$\Rightarrow (p-1)(p-9) = 0$

$\Rightarrow p = 1$ or $p = 9$

The value of p is 1 or 9. **Ans.**

Solution 9.

(b) Let the number of black balls = x

White balls = 30

Total balls = $x + 30$

$$P(\text{Black ball}) = \frac{x}{x+30}$$

$$P(\text{White ball}) = \frac{30}{x+30}$$

According to the question

$$P(\text{Black ball}) = \frac{2}{5} P(\text{White ball})$$

$$\frac{x}{x+30} = \frac{2}{5} \times \frac{30}{x+30}$$

or $\quad x = \frac{2}{5} \times 30$

$x = 12$

∴ Number of black balls = 12 **Ans.**

(c)

C.I.	f	'x' mid values	$u = \frac{x-A}{h}$	$f.u$
20—30	10	25	− 3	− 30
30—40	6	35	− 2	− 12
40—50	8	45	− 1	− 8
50—60	12	55 = A	0	0
60—70	5	65	1	5
70—80	9	75	2	18
	$\sum f = 50$			$\sum fu = -27$

Here, A = assumed mean = 55

$h = 10$

$$\overline{X} = A + \frac{\sum fu}{\sum f} h$$

$$= 55 + \frac{(-27)}{50} \times 10$$

$= 55 - 5{\cdot}4 = 49{\cdot}6$

The mean of the distribution is 49.6. **Ans.**

Solution 10.

(a) Steps of construction:

(i) 1. Draw a line BC = 6 cm

2. From B, make an angle of 120°

3. From point B, draw an arc of 3·5 cm and mark this point as A.

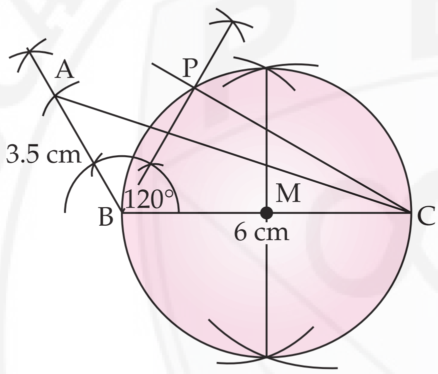
4. Join AC.

(ii) 1. Draw a perpendicular bisector on BC and mark the intersection point on BC as M.

2. Taking M as centre and BM as radius, draw a circle.

3. Draw the bisector of ∠ABC such that it touches the circle at point P.

4. Join PC.



P is the required point.

(iii) ∠ BCP = 30° **Ans.**

(b)

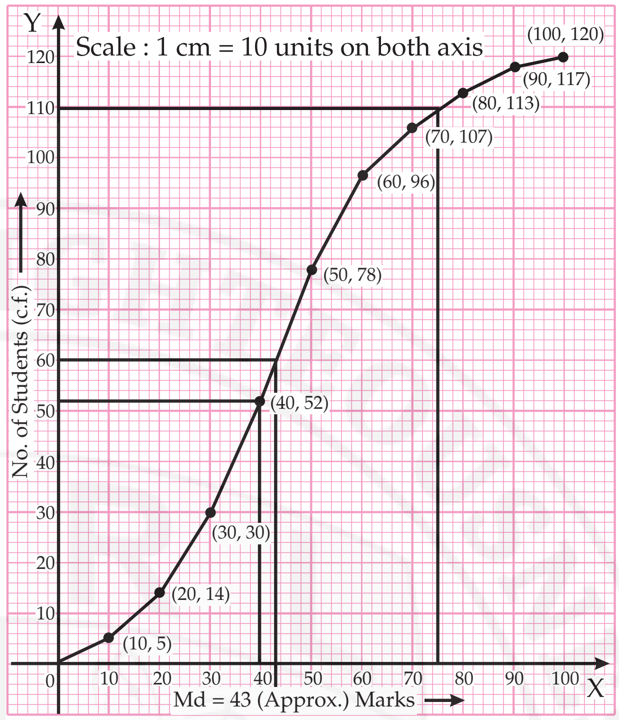
Marks C.I.	No. of Students f	$c.f.$
0—10	5	5
10—20	9	14
20—30	16	30
30—40	22	52
40—50	26	78
50—60	18	96
60—70	11	107
70—80	6	113
80—90	4	117
90—100	3	120

(i) Using graph, $n = 120$ (even)

$\therefore$ Median = $\left(\frac{120}{2}\right)^{th}$ observation

= 60^{th} observation

= 43 (approx.) **Ans.**



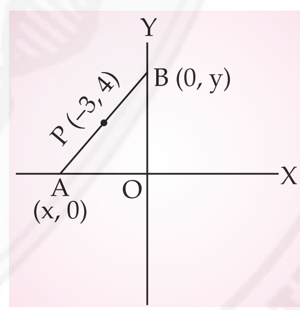
(ii) Number of students who obtained more than 75% marks in the test

= 120 – 110 = 10 **Ans.**

(iii) Number of students who did not pass the test = 52 **Ans.**

Solution 11.

(a) Given, AP : PB = 2 : 3



Let A (x, 0) and B (0, y)

$\therefore$ By section formula,

$$\frac{m_1 \times x_2 + m_2 x_1}{m_1 + m_2} = x$$

Here, $m = 2, m_2 = 3$

$x_1 = 0, x_2 = x$

$x = -3$

$$\Rightarrow \quad \frac{2 \times 0 + 3 \times x}{2 + 3} = -3$$

$\Rightarrow \quad 3x = -15$

$x = -5$

and $$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

Here, $m_1 = 2, m_2 = 3$

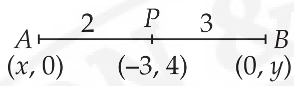
$y_1 = y, y_2 = 0$

$y = 4$

$$\Rightarrow \quad \frac{2 \times y + 3 \times 0}{2+3} = 4$$

$$\Rightarrow \quad 2y = 20$$

$$\Rightarrow \quad y = 10$$



$\therefore$ Coordinates of

$A \equiv (x, 0) \equiv (-5, 0)$

and $B \equiv (0, y) \equiv (0, 10)$ **Ans.**

(b) Given, $\frac{x^4+1}{2x^2} = \frac{17}{8}$

Using componendo and dividendo

$$\frac{x^4+1+2x^2}{x^4+1-2x^2} = \frac{17+8}{17-8}$$

$$\Rightarrow \quad \frac{(x^2+1)^2}{(x^2-1)^2} = \frac{25}{9}$$

$[\because (a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2 = a^2 - 2ab - b^2]$

$$\Rightarrow \quad \frac{x^2+1}{x^2-1} = \frac{5}{3}$$

(Taking square root on both the sides)

Again applying componendo and dividendo

$$\frac{x^2+1+x^2-1}{x^2+1-x^2+1} = \frac{5+3}{5-3}$$

$$\Rightarrow \quad \frac{2x^2}{2} = \frac{8}{2}$$

$$\Rightarrow \quad x^2 = 4$$

$$\Rightarrow \quad x = \pm 2$$

$\therefore$ Value of x is 2, –2. **Ans.**

(c) Let, the original cost of each book be ₹ x.

$\therefore$ Number of books purchased for ₹ 960 = $\frac{960}{x}$

Now, if cost of each book = ₹ $(x - 8)$

$\therefore$ Number of books purchased for ₹ 960

$= \frac{960}{x-8}$

According to the question

$$\frac{960}{x} + 4 = \frac{960}{x-8}$$

$$\Rightarrow \quad \frac{960}{(x-8)} - \frac{960}{x} = 4$$

$$\Rightarrow \quad \frac{960x - 960x + 7,680}{x(x-8)} = 4$$

$$\Rightarrow \quad 7,680 = 4x^2 - 32x$$

$$\Rightarrow \quad x^2 - 8x - 1,920 = 0$$

$$\Rightarrow \quad x^2 + 40x - 48x - 1,920 = 0$$

$$\Rightarrow \quad x(x + 40) - 48(x + 40) = 0$$

$$\Rightarrow \quad (x + 40)(x - 48) = 0$$

$$\Rightarrow \quad x = -40, 48$$

As cost can't be negative

$\therefore \quad x =$ ₹ 48.

$\therefore$ Cost of each book is ₹ 48. **Ans.**

MATHEMATICS 2012

QUESTIONS

SECTION—A (40 Marks)

*(Attempt **all** questions from this Section)*

Question 1.

(a) *If* $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ *and* $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, *find* $A^2 - 5A + 7I$. **[3]**

(b) *The monthly pocket money of Ravi and Sanjeev are in the ratio 5 : 7. Their expenditures are in the ratio 3 : 5. If each saves ₹ 80 every month, find their monthly pocket money.* **[3]**

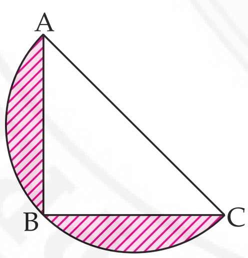
(c) *Using the Remainder Theorem, factorise completely the following polynomial.*

$$3x^3 + 2x^2 - 19x + 6$$ **[4]**

Question 2.

(a) *On what sum of money will the difference between the compound interest and simple interest for 2 years be equal to ₹ 25 if the rate of interest charged for both is 5% p.a. ?*** **[3]**

(b) *ABC is an isosceles right angled triangle with* $\angle ABC = 90°$. *A semi-circle is drawn with AC as the diameter. If AB = BC = 7 cm, find the area of the shaded region.*** $\left(\text{Take } \pi = \frac{22}{7}\right)$ **[3]**



(c) *Given a line segment AB joining the points A (– 4, 6) and B (8, – 3). Find :*

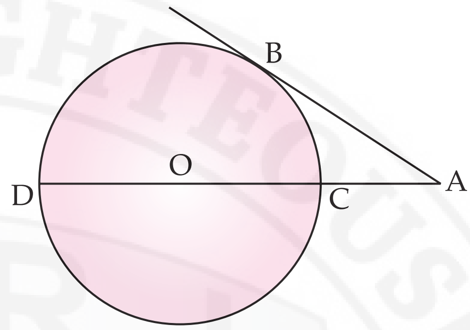
(i) *the ratio in which AB is divided by the Y-axis.*

(ii) *find the coordinates of the point of intersection.*

(iii) *the length of AB.*** **[4]**

Question 3.

(a) *In the given figure O is the centre of the circle and AB is a tangent at B. If AB = 15 cm and AC = 7.5 cm. Calculate the radius of the circle.* **[3]**



(b) *Evaluate without using trigonometric tables :***

$$\cos^2 26° + \cos 64° \sin 26° + \frac{\tan 36°}{\cot 54°}$$ **[3]**

(c) *Marks obtained by 40 students in a short assessment is given below, where a and b are two missing data.*

Marks	5	6	7	8	9
No. of Students	6	*a*	16	13	*b*

If the mean of the distribution is 7.2, find a and b. **[4]**

Question 4.

(a) *Kiran deposited ₹ 200 per month for 36 months in a bank's recurring deposit account. If the bank pays interest at the rate of 11% per annum, find the amount she gets on maturity.* **[3]**

(b) *Two coins are tossed once. Find the probability of getting :*

(i) *2 heads,*

(ii) *at least 1 tail.* **[3]**

(c) *Using graph paper and taking 1 cm = 1 unit along both X-axis and Y-axis.*

(i) *Plot the points A (– 4, 4) and B (2, 2).*

(ii) *Reflect A and B in the origin to get the images A′ and B′ respectively.*

(iii) *Write down the coordinates of A′ and B′.*

(iv) *Give the geometrical name for the figure ABA′B′.*

(v) *Draw and name its lines of symmetry.*** **[4]**

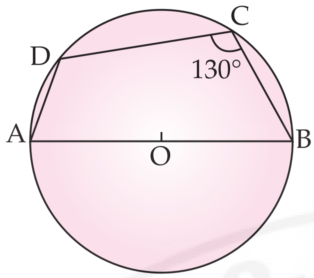
SECTION—B (40 Marks)

*(Attempt any **four** questions from this Section)*

Question 5.

(a) *In the given figure, AB is the diameter of a circle with centre O.*

** Answer is not given due to change in the present syllabus.



$\angle BCD = 130°$. Find :

(i) $\angle DAB$

(ii) $\angle DBA$ **[3]**

(b) Given $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} X = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$. Write :

(i) the order of the matrix X.

(ii) the matrix X. **[3]**

(c) A page from the Savings Bank Account of Mr. Prateek is given below :

Date	Particulars	Withdrawal (in ₹)	Deposit (in ₹)	Balances (in ₹)
January 1st 2006	B/F	—	—	1,270
January 7th 2006	By Cheque	—	2,310	3,580
March 9th 2006	To Self	2,000	—	1,580
March 26th 2006	By Cash	—	6,200	7,780
June 10th 2006	To Cheque	4,500	—	3,280
July 15th 2006	By Clearing	—	2,630	5,910
October 18th 2006	To Cheque	530	—	5,380
October 27th 2006	To Self	2,690	—	2,690
November 3rd 2006	By Cash	—	1,500	4,190
December 6th 2006	To Cheque	950	—	3,240
December 23rd 2006	By Transfer	—	2,920	6,160

If he receives ₹ 198 as interest on 1st January, 2007, find the rate of interest paid by the bank.** **[4]**

Question 6.

(a) The printed price of an article is ₹ 60,000. The wholesaler allows a discount of 20% to the shopkeeper. The shopkeeper sells the article to the customer at the printed price. Sales tax (under VAT) is charged at the rate of 6% at every stage. Find :**

(i) the cost to the shopkeeper inclusive of tax.

(ii) VAT paid by the shopkeeper to the Government.

(iii) the cost to the customer inclusive of tax. **[3]**

(b) Solve the following inequation and represent the solution set on the number line :

$$4x - 19 < \frac{3x}{5} - 2 \leq \frac{-2}{5} + x, x \in R, \quad \textbf{[3]}$$

Where R is a set of real numbers.

(c) Without solving the following quadratic equation, find the value of 'm' for which the given equation has real and equal roots.

$$x^2 + 2(m - 1)x + (m + 5) = 0 \quad \textbf{[4]}$$

Question 7.

(a) A hollow sphere of internal and external radii 6 cm and 8 cm respectively is melted and recast into small cones of base radius 2 cm and height 8 cm. Find the number of cones. **[3]**

** Answer is not given due to change in the present syllabus.

(b) Solve the following equation and give your answer correct to 3 significant figures :

$$5x^2 - 3x - 4 = 0 \quad \textbf{[3]}$$

(c) As observed from the top of a 80 m tall lighthouse, the angles of depression of two ships on the same side of the light house in horizontal line with its base are 30° and 40° respectively. Find the distance between the two ships. Give your answer correct to the nearest metre. **[4]**

Question 8.

(a) A man invests ₹ 9600 on ₹ 100 shares at ₹ 80. If the company pays him 18% dividend find:

(i) the number of shares he buys.

(ii) his total dividend.

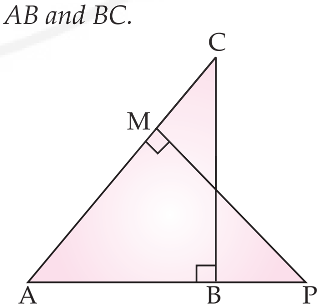
(iii) his percentage return on the shares. **[3]**

(b) In the given figure ΔABC and ΔAMP are right angled at B and M respectively.

Given AC = 10 cm, AP = 15 cm and PM = 12 cm.

(i) Prove $\Delta ABC \sim \Delta AMP$.

(ii) Find AB and BC. **[3]**



(c) If $x = \dfrac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$, using properties of proportion show that

$$x^2 - 2ax + 1 = 0$$ **[4]**

Question 9.

(a) The line through A (– 2, 3) and B (4, b) is perpendicular to the line $2x - 4y = 5$. Find the value of b. **[3]**

(b) Prove that $\dfrac{\tan^2\theta}{(\sec\theta - 1)^2} = \dfrac{1+\cos\theta}{1-\cos\theta}$. **[3]**

(c) A car covers a distance of 400 km at a certain speed. Had the speed been 12 km/h more, the time taken for the journey would have been 1 hour 40 minutes less. Find the original speed of the car. **[4]**

Question 10.

(a) Construct a triangle ABC in which base BC = 6 cm, AB = 5.5 cm and ∠ ABC = 120°.

(i) Construct a circle circumscribing the triangle ABC.

(ii) Draw a cyclic quadrilateral ABCD so that D is equidistant from B and C. **[4]**

(b) The following distribution represents the height of 160 students of a school.

Height (in cm)	No. of Students
140–145	12
145–150	20
150–155	30
155–160	38
160–165	24
165–170	16
170–175	12
175–180	8

Draw an ogive for the given distribution taking 2 cm = 5 cm of height on one axis and 2 cm = 20 students on the other axis. Using the graph, determine :

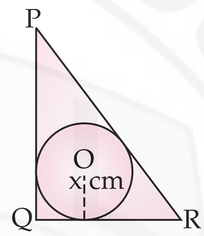
(i) The median height.

(ii) The interquartile range.

(iii) The number of students whose height is above 172 cm. **[6]**

Question 11.

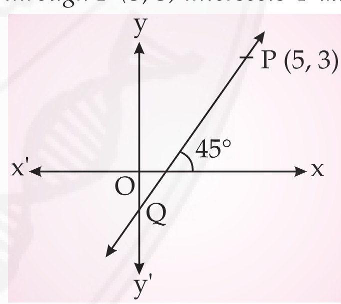
(a) In triangle PQR, PQ = 24 cm, QR = 7 cm and ∠ PQR = 90°. Find the radius of the inscribed circle. **[3]**



(b) Find the mode and median of the following frequency distribution : **[3]**

x	10	11	12	13	14	15
f	1	4	7	5	9	3

(c) The line through P (5, 3) intersects Y-axis at Q.



(i) Write the slope of the line.

(ii) Write the equation of the line.

(iii) Find the coordinates of Q. **[4]**

Answers

SECTION—A

Solution 1.

(a)

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$$

Now, $$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15 & 5-5 \\ -5+5 & 3-10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$ **Ans.**

(b) Let, the monthly pocket money of Ravi and Sanjeev be $5x$ and $7x$ respectively and their expenditures be $3y$ and $5y$.

So, $5x - 3y = 80$ ...(i)

And, $7x - 5y = 80$ ...(ii)

Multiplying equation (i) by 5 and equation (ii) by 3, we get

$25x - 15y = 400$ ...(iii)

$21x - 15y = 240$ ...(iv)

Sub. $\quad - \quad + \quad -$

$4x = 160$

$\Rightarrow \quad x = 40$

So monthly pocket money of Ravi

$= ₹\ 5 \times 40 = ₹\ 200$

And of Sanjeev $= ₹\ 7 \times 40 = ₹\ 280$ **Ans.**

(c) Let, $P(x) = 3x^3 + 2x^2 - 19x + 6$

Putting $x = 2$, $P(2) = 3 \times 2^3 + 2 \times 2^2 - 19 \times 2 + 6$

$= 24 + 8 - 38 + 6$

$= 38 - 38 = 0$

$\Rightarrow (x - 2)$ is a factor of P (x)

Now,

$$
\begin{array}{r|l}
 & 3x^2 + 8x - 3 \\
x - 2 & 3x^3 + 2x^2 - 19x + 6 \\
 & 3x^3 - 6x^2 \\
 & - \quad + \\
 & 8x^2 - 19x \\
 & 8x^2 - 16x \\
 & - \quad + \\
 & -3x + 6 \\
 & -3x + 6 \\
 & + \quad - \\
 & 0
\end{array}
$$

$\Rightarrow 3x^3 + 2x^2 - 19x + 6 = (x - 2) \cdot (3x^2 + 8x - 3)$

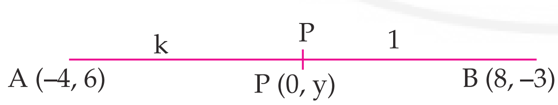
$= (x - 2)(3x^2 + 9x - x - 3)$

$= (x-2)[3x(x+3) - 1(x+3)]$

$= (x - 2)(x + 3)(3x - 1)$ **Ans.**

Solution 2.

(c)



Let the line segment AB is divided by Y-axis at point P in the ratio $k : 1$.

(i) Since P lies on Y-axis so $x = 0$, then coordinates of P are $(0, y)$.

We have, $x = \dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}$

$\Rightarrow \quad 0 = \dfrac{k \times 8 + 1 \times (-4)}{k + 1}$

$\Rightarrow \quad 8k - 4 = 0$

$\Rightarrow \quad 8k = 4$

$\Rightarrow \quad k = \dfrac{4}{8} = \dfrac{1}{2}$

So, required ratio is 1 : 2 **Ans.**

(ii) Now, $y = \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}$

$y = \dfrac{1 \times (-3) + 2 \times 6}{1 + 2}$

$= \dfrac{-3 + 12}{3} = 3$

So, coordinates of point of intersection on Y-axis are (0, 3). **Ans.**

Solution 3.

(a) Given, AB = 15 cm, AC = 7·5 cm.

If a chord and a tangent intersect externally then product of segments of the chord is equal to square of the length of the tangent.

$AB^2 = AC \times AD$

$\Rightarrow \quad 15^2 = 7{\cdot}5 \times AD$

$\Rightarrow \quad AD = \dfrac{225}{7{\cdot}5} = 30$

$\Rightarrow \quad CD = AD - AC$

$= 30 - 7{\cdot}5 = 22{\cdot}5$

Radius $= \dfrac{1}{2} \times CD$

Radius $= \dfrac{1}{2} \times 22{\cdot}5$

Radius = 11·25 cm. **Ans.**

(c)

Marks (x)	No. of students (f)	$f \cdot x$
5	6	30
6	a	$6a$
7	16	112
8	13	104
9	b	$9b$
	$\Sigma f = 35 + a + b$	$\Sigma fx = 246 + 6a + 9b$

Now,

$35 + a + b = 40$

$a + b = 5$ ...(i)

And, $\overline{X} = \dfrac{\Sigma fx}{\Sigma f}$

$7{\cdot}2 = \dfrac{246 + 6a + 9b}{40}$

$\Rightarrow \quad 6a + 9b + 246 = 288$

$\Rightarrow \quad 6a + 9b = 42$

$\Rightarrow \quad 2a + 3b = 14 \quad \ldots(ii)$

Multiplying by 2 in equation (i) and solving with equation (ii)

$2a + 2b = 10$

$2a + 3b = 14$

On subtracting (–) (–) (–)

$-b = -4$

$\Rightarrow \quad b = 4$

Putting the value of b in equation (i) , we get

$a + 4 = 5$

$\Rightarrow \quad a = 1$

$\therefore \quad a = 1, \; b = 4$ **Ans.**

Solution 4.

(a) Given, P = ₹ 200, n = 36 months, R = 11%

$$\text{Interest} = \frac{P \times n(n+1) \times R}{2 \times 12 \times 100}$$

$$= \frac{200 \times 36 \times 37 \times 11}{2{,}400}$$

$= 3 \times 37 \times 11 =$ ₹ 1,221

Sum deposited $= n \times P$

$= 36 \times 200 =$ ₹ 7,200

$\Rightarrow$ Amount $= nP + I$

$= 7{,}200 + 1{,}221$

= ₹ 8,421

$\therefore$ Total amount she will get is ₹ 8,421. **Ans.**

(b) If two coins are tossed once, then total outcomes

S = {HH, HT, TH, TT}

$\Rightarrow \quad n(S) = 4$

(i) Let E be the event of getting two heads

E = {HH}

$\therefore$ Favourable outcomes

$n(E) = 1$

Required probability

$$P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{1}{4}$$ **Ans.**

(ii) Let F be the event of getting atleast one tail

(F) = {HT, TH, TT}

$\therefore$ Favourable outcomes

$n(F) = 3$

Required probability

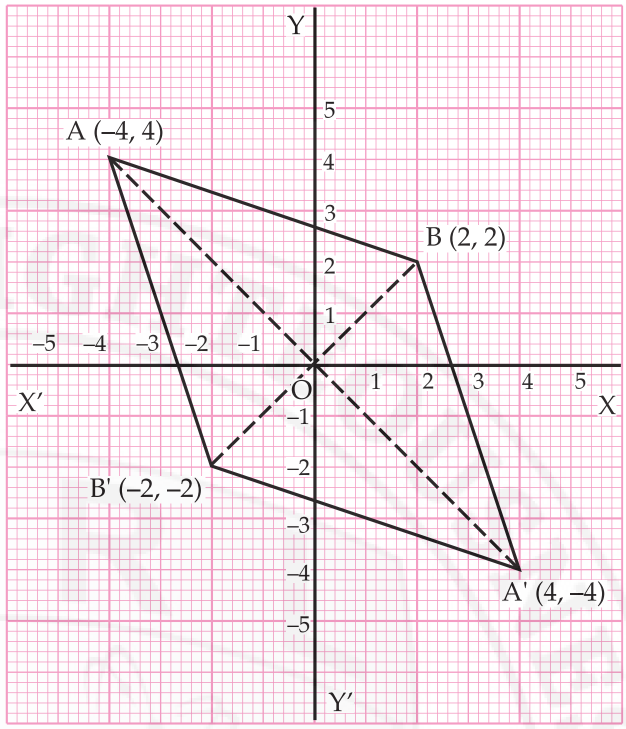
$$P(F) = \frac{n(F)}{n(S)} = \frac{3}{4}$$ **Ans.**

(c) **(i), (ii)** on graph

(iii) A′ (4, – 4)

B′ (– 2, – 2)

(iv) Rhombus

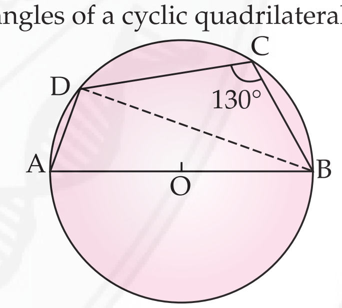


SECTION—B

Solution 5.

(a) **(i)** $\angle DAB + \angle BCD = 180°$

(Opp. angles of a cyclic quadrilateral)



$\Rightarrow \quad \angle DAB + 130° = 180°$

($\angle BCD = 130°$ given)

$\Rightarrow \quad \angle DAB = 180° - 130°$

$\Rightarrow \quad \angle DAB = 50°$ **Ans.**

(ii) $\angle ADB = 90°$

(angle in semi-circle)

In Δ ADB,

$\angle DAB + \angle ADB + \angle DBA = 180°$

(Angle sum property)

$\Rightarrow \quad 50° + 90° + \angle DBA = 180°$

$\Rightarrow \quad \angle DBA = 180° - 140°$

$\Rightarrow \quad \angle DBA = 40°$ **Ans.**

(b) **(i)** $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}_{2 \times 2} X = \begin{bmatrix} 7 \\ 6 \end{bmatrix}_{2\times 1}$

According to the given condition, the order of matrix X will be 2 × 1. **Ans.**

(ii) Let $X = \begin{bmatrix} a \\ b \end{bmatrix}$

so $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a+b \\ -3a+4b \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$

$\Rightarrow 2a + b = 7$ ...(i)

$\Rightarrow -3a + 4b = 6$ ...(ii)

Multiplying by 4 in equation (i) and solving with equation (ii)

$8a + 4b = 28$

$-3a + 4b = 6$

On subtracting (+) (–) (–)

$11a = 22$

$\therefore a = 2$

Putting the value of a in equation (i) , we get

$2 \times 2 + b = 7$

$\therefore b = 7 - 4 = 3$

$\Rightarrow X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ **Ans.**

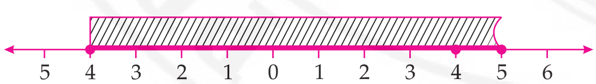
Solution 6.

(b) $4x - 19 < \frac{3x}{5} - 2 \le \frac{-2}{5} + x, \; x \in R$

$\Rightarrow 4x - 19 < \frac{3x}{5} - 2$	$\frac{3x}{5} - 2 \le \frac{-2}{5} + x$
$4x - \frac{3x}{5} < -2 + 19$	$\frac{3x}{5} - x \le \frac{-2}{5} + 2$
$\Rightarrow \frac{17x}{5} < 17$	$-2x \le 8$
$\Rightarrow x < 5$	$2x \ge -8$
	$x \ge -4$

$\Rightarrow -4 \le x < 5$

Solution Set : $\{x : -4 \le x < 5, x \in R\}$



(c) Given quadratic equation

$x^2 + 2(m-1)x + (m+5) = 0$

On comparing with $ax^2 + bx + c = 0$

$a = 1, b = 2(m-1), \; c = (m+5)$

Since equation has real and equal roots

$\therefore D = 0$

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow [2(m-1)]^2 - 4 \times 1 \times (m+5) = 0$

$\Rightarrow 4(m-1)^2 - 4(m+5) = 0$

$\Rightarrow 4[(m-1)^2 - (m+5)] = 0$

$\Rightarrow 4[m^2 - 2m + 1 - m - 5] = 0$

$\Rightarrow m^2 - 3m - 4 = 0$

$\Rightarrow m^2 - 4m + m - 4 = 0$

$\Rightarrow m(m-4) + 1(m-4) = 0$

$\Rightarrow (m+1)(m-4) = 0$

$m + 1 = 0, \quad m - 4 = 0$

$\Rightarrow m = -1$ and $m = 4$

$\therefore m = -1, 4$ **Ans.**

Solution 7.

(a) Volume of metal in hollow sphere

$= \frac{4}{3}\pi(8^3 - 6^3)$

$= \frac{1184}{3}\pi \text{ cm}^3$

Volume of metal in one cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 2^2 \times 8$

$= \frac{32}{3}\pi \text{ cm}^3$

Number of cones $= \frac{\text{Volume of metal in sphere}}{\text{Volume of metal in one cone}}$

$= \frac{\frac{1184}{3}\pi}{\frac{32}{3}\pi}$

$= \frac{1184}{32} = 37$ **Ans.**

(b) Given equation is, $5x^2 - 3x - 4 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$a = 5, b = -3, c = -4$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{3 \pm \sqrt{9 - 4 \times 5(-4)}}{2 \times 5}$

$\Rightarrow x = \frac{3 \pm \sqrt{9 + 80}}{10} = \frac{3 \pm \sqrt{89}}{10}$

$x = \frac{3 \pm 9{\cdot}434}{10}$

$\Rightarrow x = \frac{3 + 9{\cdot}434}{10}$

or $x = \frac{3 - 9.434}{10}$

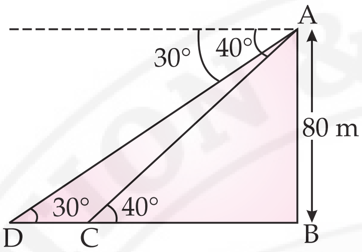
$x = \frac{12{\cdot}434}{10}$

or $\qquad x = \dfrac{-6{\cdot}434}{10}$

$\qquad x = 1{\cdot}243$

or $\qquad x = -0.643$ **Ans.**

(c) In fig. AB is 80 m tall light house, the two ships are at C and D.



In Δ ABC,

$$\tan 40° = \frac{AB}{BC}$$

$$\Rightarrow \quad BC = \frac{AB}{\tan 40°}$$

$$BC = \frac{80}{0{\cdot}8391}$$

(using trigonometric table)

$$= 95{\cdot}34 \text{ m}$$

In Δ ABD,

$$\tan 30° = \frac{AB}{BD}$$

$$\Rightarrow \quad BD = \frac{AB}{\tan 30°} = \frac{80}{0{\cdot}5774}$$

$$= 138{\cdot}55 \text{ m}$$

Distance between two ships

$$DC = BD - BC$$

$$= 138{\cdot}55 - 95{\cdot}34$$

$$= 43{\cdot}21 \text{ m} = 43 \text{ m (approx.)}$$

Ans.

Solution 8.

(a) (i) Number of shares $= \dfrac{\text{Investment}}{\text{M.V.}}$

$$= \frac{9{,}600}{80} = 120$$ **Ans.**

(ii) Total dividend $= \dfrac{18}{100} \times 100 \times 120$

$$= ₹\ 2{,}160$$ **Ans.**

(iii) Percentage return

$$= \frac{\text{Total divided}}{\text{Investment}}$$

$$= \frac{2{,}160}{9{,}600} \times 100 = 22{\cdot}5\%$$

Ans.

(b) Given, Δ ABC and Δ AMP, with right angle at B and M respectively. AC = 10 cm, AP = 15 cm, and PM = 12 cm.

(i) In Δ ABC and Δ AMP

$\angle ABC = \angle AMP$ (90° each)

$\angle A = \angle A$ (Common)

$\therefore \quad \Delta ABC \sim \Delta AMP$ (By AA similarity)

Hence Proved.

(ii) Given, AC = 10 cm, AP = 15 cm, PM = 12 cm.

$\because \quad \Delta ABC \sim \Delta AMP$

$$\therefore \quad \frac{AB}{AM} = \frac{BC}{PM} = \frac{AC}{AP}$$

(corresponding sides of similar triangles are in proportion)

$$\Rightarrow \quad \frac{BC}{PM} = \frac{AC}{AP}$$

$$\Rightarrow \quad \frac{BC}{12} = \frac{10}{15}$$

$$\Rightarrow \quad BC = \frac{10}{15} \times 12$$

$$BC = 8 \text{ cm}$$ **Ans.**

In Δ ABC, right angled at B

$$AC^2 = AB^2 + BC^2$$

(Pythagoras theorem)

Now, $\quad AB^2 = AC^2 - BC^2$

$$= 10^2 - 8^2 = 100 - 64 = 36$$

$$AB = 6 \text{ cm}$$ **Ans.**

(c) Given, $\quad x = \dfrac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$

$$\Rightarrow \quad \frac{x}{1} = \frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$$

Using componendo and dividendo

$$\Rightarrow \quad \frac{x+1}{x-1} = \frac{\sqrt{a+1}+\sqrt{a-1}+\sqrt{a+1}-\sqrt{a-1}}{\sqrt{a+1}+\sqrt{a-1}-\sqrt{a+1}+\sqrt{a-1}}$$

$$\Rightarrow \quad \frac{x+1}{x-1} = \frac{2\sqrt{a+1}}{2\sqrt{a-1}}$$

$$\Rightarrow \quad \frac{(x+1)^2}{(x-1)^2} = \frac{a+1}{a-1}$$

(Squaring both side)

$$\Rightarrow \quad \frac{x^2+2x+1}{x^2-2x+1} = \frac{a+1}{a-1}$$

Again using componendo and dividendo

$$\frac{(x^2+2x+1)+(x^2-2x+1)}{(x^2+2x+1)-(x^2-2x+1)} = \frac{(a+1)+(a-1)}{(a+1)-(a-1)}$$

$$\Rightarrow \quad \frac{x^2+2x+1+x^2-2x+1}{x^2+2x+1-x^2+2x-1} = \frac{a+1+a-1}{a+1-a+1}$$

$$\Rightarrow \quad \frac{2x^2+2}{4x} = \frac{2a}{2}$$

$$\Rightarrow \quad \frac{x^2+1}{2x} = \frac{a}{1}$$

$$\Rightarrow \quad x^2 + 1 = 2ax$$

$$\Rightarrow \quad x^2 - 2ax + 1 = 0.$$ **Hence Proved.**

Solution 9.

(a) Given, A (– 2, 3), B (4, b)

$$\text{Slope of AB} = \frac{y_2-y_1}{x_2-x_1}$$

$$\Rightarrow \quad m_1 = \frac{b-3}{4+2}$$

$$\Rightarrow \quad m_1 = \frac{b-3}{6}$$

And $2x - 4y = 5$

$$\Rightarrow \quad 4y = 2x - 5$$

$$\Rightarrow \quad y = \frac{1}{2}x - \frac{5}{4}$$

On comparing with $y = mx + c$

$$\text{Slope } (m_2) = \frac{1}{2}$$

Since both lines are perpendicular to each other

$$\therefore \quad m_1 \times m_2 = -1$$

$$\frac{b-3}{6} \times \frac{1}{2} = -1$$

$$b - 3 = -12$$

$$b = -9$$ **Ans.**

(b)
$$\text{L.H.S.} = \frac{\tan^2\theta}{(\sec\theta-1)^2}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\left(\frac{1}{\cos\theta}-1\right)^2}$$

$$\left(\because \tan\theta = \frac{\sin\theta}{\cos\theta};\ \sec\theta = \frac{1}{\cos\theta}\right)$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{(1-\cos\theta)^2}{\cos^2\theta}} = \frac{\sin^2\theta}{(1-\cos\theta)^2}$$

$$= \frac{1-\cos^2\theta}{(1-\cos\theta)^2} \qquad (\because \sin^2\theta = 1-\cos^2\theta)$$

$$= \frac{(1-\cos\theta)(1+\cos\theta)}{(1-\cos\theta)^2}$$

$$[\because a^2 - b^2 = (a-b)(a+b)]$$

$$= \frac{1+\cos\theta}{1-\cos\theta} = \text{R.H.S.}$$ **Hence Proved.**

(c) Let the original speed of the car be x km/h.

So, time taken by car $= \frac{400}{x}$ hrs

When, Speed $= (x + 12)$ km/h

Time taken by car $= \frac{400}{x+12}$ hrs

According to the question,

$$\frac{400}{x} - \frac{400}{x+12} = 1 \text{ hour} + 40 \text{ minutes}$$

$$\left[\left(1+\frac{40}{60}\right)\text{hour}\right]$$

$$400\left[\frac{(x+12-x)}{x(x+12)}\right] = 1 + \frac{2}{3}$$

$$\Rightarrow \quad \frac{4800}{x^2+12x} = \frac{5}{3}$$

$$\Rightarrow \quad 5(x^2 + 12x) = 14{,}400$$

$$\Rightarrow \quad x^2 + 12x - 2{,}880 = 0$$

$$\Rightarrow \quad x^2 + 60x - 48x - 2{,}880 = 0$$

$$\Rightarrow \quad x(x + 60) - 48(x + 60) = 0$$

$$\Rightarrow \quad (x + 60)(x - 48) = 0$$

Either, $x + 60 = 0$

$x = -60$

(Neglect, speed can't be negative)

or $x - 48 = 0$

$x = 48$

Hence, original speed of the car = 48 km/h

Ans.

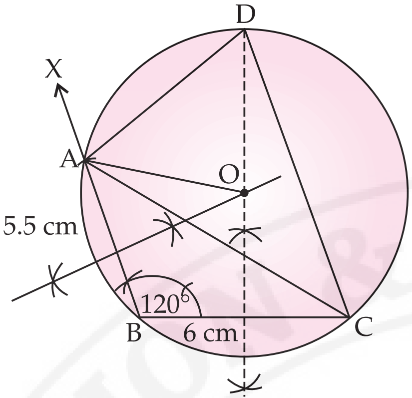
Solution 10.

(a) **(i)** Steps of construction :

1. Draw a line segment BC = 6 cm.
2. Construct ∠ XBC = 120°.
3. From B, cut an arc of 5·5 cm on side XB, and mark this point as A.
4. Join A to C.
5. Construct perpendicular bisectors of AB and BC, intersecting at O. Join AO.
6. Taking O as centre and OA as radius, draw a circle, passing through A, B and C.

(ii) 1. Extend the right bisector of BC, intersecting the circle at D.

2. Join A to D and C to D.

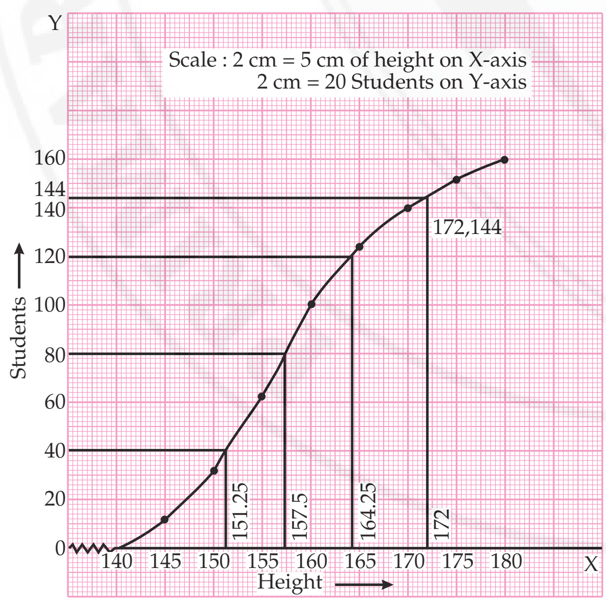


$\therefore$ ABCD is required cyclic quadrilateral.

(b)

Height (in cm)	No. of Students (*f*)	*cf*
140–145	12	12
145–150	20	32
150–155	30	62
155–160	38	100
160–165	24	124
165–170	16	140
170–175	12	152
175–180	8	160
	$\sum f = 160$	

We have to plot (145, 12), (150, 32), (155, 62), (160, 100), (165, 124), (170, 140), (175, 152) and (180, 160).



(i) Using graph,

$$N = 160 \text{ (even)}$$

$$\therefore \quad \text{Median} = \left(\frac{160}{2}\right)^{th} \text{ term} = 80^{th} \text{ term}$$

Now, we shall construct a horizontal line at cumulative frequency = 80 :

Intersecting the ogive at (157·5, 80),

Hence, median height = 157·5 cm. **Ans.**

(ii) Lower quartile $(Q_1) = \left(\frac{N}{4}\right)^{th}$ term

$= \left(\frac{160}{4}\right)^{th}$ term

$= 40^{th}$ term $= 151{\cdot}25$

Upper quartile $(Q_3) = \left(\frac{3N}{4}\right)^{th}$ term

$= \left(\frac{3 \times 160}{4}\right)^{th}$ term

$= 120^{th}$ term $= 164{\cdot}25$

$\therefore$ Interquartile range $= Q_3 - Q_1$

$= 164{\cdot}25 - 151{\cdot}25$

$= 13$ **Ans.**

(iii) The number of students whose height is above 172 cm

$= 160 - 144 = 16$ **Ans.**

Solution 11.

(a) Given, PQ = 24 cm, QR = 7 cm, and $\angle PQR = 90°$.

Construction : Draw $OM \perp QR$ and $ON \perp PQ$

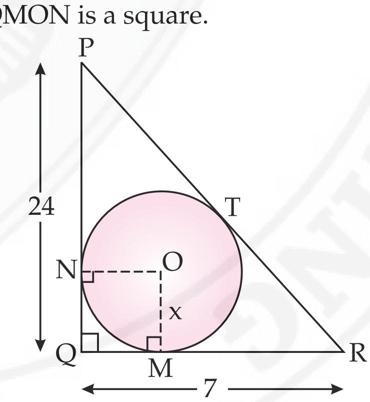
As, $OM \perp QR$ and $ON \perp PQ$:

(Tangents and radius are perpendicular to each other)

and OM = ON (Radius)

and QM = QN (Tangents from an external point)

$\therefore$ QMON is a square.



$\Rightarrow$ QM = OM = ON = QN = x cm (say)

So, MR = $(7 - x)$ cm

PN = $(24 - x)$ cm

PT = PN = $24 - x$

and, MR = RT = $7 - x$

(Tangents from an external point)

$\Rightarrow$ PR = PT + RT

$= 24 - x + 7 - x = 31 - 2x$

PQ = 24 cm, QR = 7 cm, PQR = 90° (Given)

Now, in Δ PQR

$$PR^2 = PQ^2 + QR^2$$

(by Pythagoras theorem)

$$= 24^2 + 7^2$$

$$= 576 + 49 = 625$$

$\Rightarrow$ PR = 25 cm

$\Rightarrow$ $31 - 2x = 25$

$\Rightarrow$ $2x = 31 - 25$

$\Rightarrow$ $2x = 6$

$\Rightarrow$ $x = 3$ cm

∴ Radius of the inscribed circle is 3 cm. **Ans.**

(b)

x	f	cf
10	1	1
11	4	5
12	7	12
13	5	17
14	9	26
15	3	29

$\Rightarrow$ Mode = 14

(Since 9 is highest frequency)

Now, N = 29 (odd)

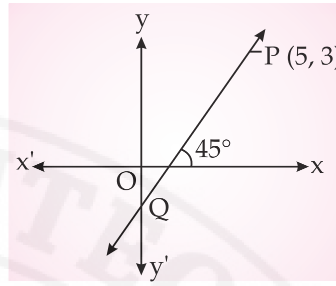
$$\therefore \quad \text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{value}$$

$$= \left(\frac{29+1}{2}\right)^{\text{th}} \text{value}$$

= 15th value = 13

∴ Mode = 14 and Median = 13. **Ans.**

(c) (i) $m = \tan\theta = \tan 45°$



∴ Slope of the line

$m = 1$ **Ans.**

(ii) Equation of line PQ,

where $x_1 = 5$, $y_1 = 3$ and $m = 1$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 1(x - 5)$$

$$y - 3 = x - 5$$

$\Rightarrow$ $x - y - 2 = 0$ **Ans.**

(iii) Equation of line PQ is

$$x - y - 2 = 0$$

Put $x = 0$

[Since, at Q coordinates are $(0, y)$]

$$-y - 2 = 0$$

$\Rightarrow$ $y = -2$

So, coordinates of Q (0, – 2). **Ans.**

MATHEMATICS 2011

QUESTIONS

SECTION—A (40 Marks)

*(Attempt **all** questions from this Section)*

Question 1.

(a) *Find the value of 'k' if (x – 2) is a factor of :*

$$x^3 + 2x^2 - kx + 10$$

Hence, determine whether (x + 5) is also a factor. **[3]**

(b) *If* $A = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}$ *and* $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, *is the product AB possible ? Give a reason. If yes, find AB.* **[3]**

(c) *Mr. Kumar borrowed ₹ 15,000 for two years. The rate of interest for the two successive years are 8% and 10% respectively. If he repays ₹ 6,200 at the end of the first year, find the outstanding amount at the end of the second year.*** **[4]**

Question 2.

(a) *From a pack of 52 playing cards all cards whose numbers are multiples of 3 are removed. A card is now drawn at random.*

What is the probability that the card drawn is :

(i) *a face card (King, Jack or Queen)*

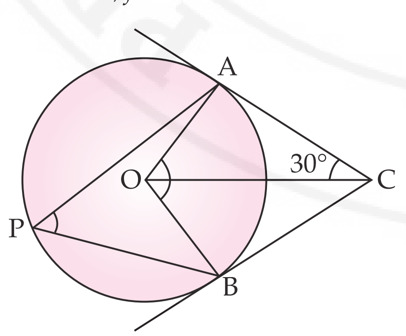
(ii) *an even numbered red card ?* **[3]**

(b) *Solve the following equation :*

$x - \frac{18}{x} = 6$. *Give your answer correct to two significant figures.* **[3]**

(c) *In the given figure O is the centre of the circle. Tangents at A and B meet at C.*

If ∠ ACO = 30°, find



(i) ∠ *BCO* **(ii)** ∠ *AOB*

(iii) ∠ *APB* **[4]**

** Answer is not given due to change in the present syllabus.

Question 3.

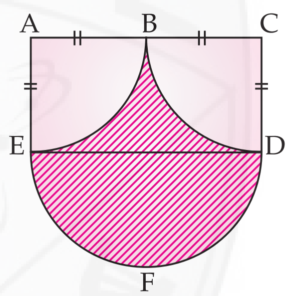
(a) *Ahmed has a recurring deposit account in a bank. He deposits ₹ 2,500 per month for 2 years. If he gets ₹ 66,250 at the time of maturity, find :*

(i) *The interest paid by the bank*

(ii) *The rate of interest.* **[3]**

(b) *Calculate the area of the shaded region, if the diameter of the semi-circle is equal to 14 cm.***

$\left(\text{Take } \pi = \frac{22}{7}\right)$ **[3]**



(c) *ABC is a triangle and G (4, 3) is the centroid of the triangle. If A = (1, 3), B = (4, b) and C = (a, 1), find 'a' and 'b'.*

Find the length of side BC. **[4]**

Question 4.

(a) *Solve the following inequation and represent the solution set on the number line* $2x - 5 \leq 5x + 4 < 11$, *where* $x \in I$, *I is a set of integers.* **[3]**

(b) *Evaluate without using trigonometric tables.***

$$2\left(\frac{\tan 35°}{\cot 55°}\right)^2 + \left(\frac{\cot 55°}{\tan 35°}\right)^2 - 3\left(\frac{\sec 40°}{\text{cosec } 50°}\right)$$ **[3]**

(c) *A Mathematics aptitude test of 50 students was recorded as follows :*

Marks	No. of Students
50-60	4
60-70	8
70-80	14
80-90	19
90-100	5

Draw a histogram for the above data using a graph paper and locate the mode. **[4]**

ICSE Mathematics-2011

SECTION—B (40 Marks)

*(Attempt any **four** questions from this Section)*

Question 5.

(a) *A manufacturer sells a washing machine to a wholesaler for ₹ 15,000. The wholesaler sells it to a trader at a profit of ₹ 1,200 and the trader in turn sells it to a consumer at a profit of ₹ 1,800. If the rate of VAT is 8% find,***

(i) *The amount of VAT received by the State Government on the sale of this machine from the manufacturer and the wholesaler.*

(ii) *The amount that the consumer pays for the machine.* **[3]**

(b) *A solid cone of radius 5 cm and height 8 cm is melted and made into small spheres of radius 0·5 cm. Find the number of spheres formed.* **[3]**

(c) *ABCD is a parallelogram where A (x, y), B (5, 8), C (4, 7) and D (2, – 4). Find*

(i) *Coordinates of A*

(ii) *Equation of diagonal BD.* **[4]**

Question 6.

(a) *Use a graph paper to answer the following questions. (Take 1 cm = 1 unit on both axes) :*

(i) *Plot A (4, 4), B (4, – 6) and C (8, 0), the vertices of a triangle ABC.*

(ii) *Reflect ABC on the Y-axis and name it as A′B′C′.*

(iii) *Write the coordinates of the images A′, B′ and C′.*

(iv) *Give a geometrical name for the figure AA′C′B′ BC.*

(v) *Identify the line of symmetry of AA′C′B′BC.*** **[5]**

(b) *Mr. Chaudhary opened a Saving's Bank Account at State Bank of India on 1st April, 2007. The entries of one year as shown in his pass book are given below :***

Date	Particulars	Withdrawals (in ₹)	Deposits (in ₹)	Balance (in ₹)
1st April, 2007	By Cash	—	8550·00	8550·00
12th April, 2007	To Self	1200·00	—	7350·00
24th April, 2007	By Cash	—	4550·00	11900·00
8th July, 2007	By Cheque	—	1500·00	13400·00
10th Sept., 2007	By Cheque	—	3500·00	16900·00
17th Sept., 2007	To Cheque	2500·00	—	14400·00
11th Oct., 2007	By Cash	—	800·00	15200·00
6th Jan., 2008	To Self	2000·00	—	13200·00
9th March, 2008	By Cheque	—	950·00	14150·00

If the bank pays interest at the rate of 5% per annum, find the interest paid on 1st April, 2008. Give your answer correct to the nearest rupee. **[5]**

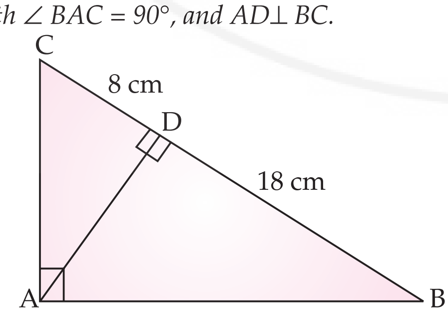
Question 7.

(a) *Using componendo and dividendo, find the value of x if*

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}} = 9.$$ **[3]**

(b) *If* $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$ *and I is the identity matrix of the same order and* A^t *is the transpose of matrix A, find* $A^t.B + BI$. **[3]**

(c) *In the following figure ABC is a right angled triangle with ∠ BAC = 90°, and AD⊥ BC.*



(i) *Prove Δ ADB ~ Δ CDA.*

(ii) *If BD = 18 cm, CD = 8 cm find AD.*

(iii) *Find the ratio of the area of Δ ADB to area of Δ CDA.* **[4]**

Question 8.

(a) **(i)** *Using step-deviation method, calculate the mean marks of the following distribution.*

(ii) *State the modal class.* **[5]**

Class interval	Frequency
50–55	5
55–60	20
60–65	10
65–70	10
70–75	9
75–80	6
80–85	12
85–90	8

** Answer is not given due to change in the present syllabus.

(b) *Marks obtained by 200 students in an examination are given below :*

Marks	Frequency
0-10	5
10-20	11
20-30	10
30-40	20
40-50	28
50-60	37
60-70	40
70-80	29
80-90	14
90-100	6

Draw an ogive for the given distribution taking 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis. Using the graph, determine :

(i) *The median marks*

(ii) *The number of students who failed if minimum marks required to pass is 40.*

(iii) *If scoring 85 and more marks is considered as grade one, find the number of students who secured grade one in the examination.* **[5]**

Question 9.

(a) *Mr. Parekh invested ₹ 52,000 on ₹ 100 shares at a discount of ₹ 20 paying 8% dividend. At the end of one year he sells the shares at a premium of ₹ 20. Find :*

(i) *The annual dividend.*

(ii) *The profit earned including his dividend.* **[3]**

(b) *Draw a circle of radius 3·5 cm. Mark a point P outside the circle at a distance of 6 cm from the centre. Construct two tangents from P to the given circle. Measure and write down the length of one tangent.* **[3]**

(c) *Prove that*

$(\operatorname{cosec} A - \sin A)(\sec A - \cos A)\sec^2 A = \tan A.$ **[4]**

Question 10.

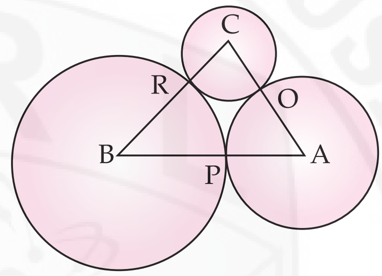
(a) *6 is the mean proportion between two numbers x and y and 48 is the third proportional of x and y. Find the numbers.* **[3]**

(b) *In what period of time will ₹ 12,000 yield ₹ 3,972 as compound interest at 10% per annum, if compounded on an yearly basis ?***** **[3]**

(c) *A man observes the angle of elevation of the top of a building to be 30°. He walks towards it in a horizontal line through its base. On covering 60 m the angle of elevation changes to 60°. Find the height of the building correct to the nearest metre.* **[4]**

Question 11.

(a) *ABC is a triangle with AB = 10 cm, BC = 8 cm and AC = 6 cm (not drawn to scale). Three circles are drawn touching each other with the vertices as their centres. Find the radii of the three circles.* **[3]**



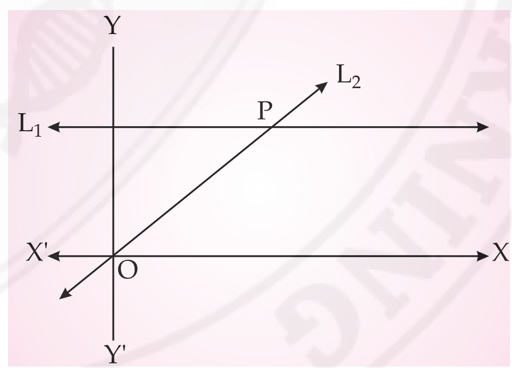
(b) *₹ 480 is divided equally among 'x' children. If the number of children were 20 more than each would have got ₹ 12 less. Find 'x'.* **[3]**

(c) *Given equation of line L_1 is y = 4.*

(i) *Write the slope of line L_2 if L_2 is the bisector of angle O.*

(ii) *Write the coordinates of point P.*

(iii) *Find the equation of L_2.* **[4]**



ANSWERS

SECTION—A

Solution 1.

(a) Let, $f(x) = x^3 + 2x^2 - kx + 10$ …(i)

As $(x - 2)$ is a factor of $f(x)$

Put $(x - 2) = 0 \Rightarrow x = 2$

$\therefore \quad f(2) = (2)^3 + 2(2)^2 - k(2) + 10$

$\Rightarrow \quad 0 = 8 + 8 - 2k + 10$

[As $(x - 2)$ is a factor of $f(x)$ $\Rightarrow f(2) = 0$]

$\Rightarrow \quad 2k = 26$

$\Rightarrow \quad k = \frac{26}{2}$

$= 13$

$\therefore \quad f(x) = x^3 + 2x^2 - 13x + 10$ …(ii)

To determine whether $(x + 5)$ is a factor of $f(x)$ or not

Put $x + 5 = 0$ *i.e.,* $x = -5$ in (ii)

We get, $f(-5) = (-5)^3 + 2(-5)^2 - 13(-5) + 10$ $[k = 13]$

$= -125 + 50 + 65 + 10 = 0$

$\therefore$ $(x + 5)$ is a factor of $f(x)$. **Ans.**

(b) $A = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}_{2\times 2}$ and $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}_{2\times 1}$

The order of matrix A is 2 × 2 and matrix B is 2 × 1.

The product AB is possible as the number of columns in A is equal to the number of rows in B.

Now, $AB = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$AB = \begin{bmatrix} 3\times 2 + 5\times 4 \\ 4\times 2 + (-2)\times 4 \end{bmatrix}$

$AB = \begin{bmatrix} 26 \\ 0 \end{bmatrix}$ **Ans.**

Solution 2.

(a) The numbers which are multiple of 3 in 52 playing cards are 3, 6 and 9 *i.e.,* 3 cards of each denomination.

$\therefore$ All cards whose numbers are multiples of 3 are

$= 4 \times 3 = 12$ cards

Remaining cards = 52 – 12 = 40

[Jack, Queen and King of each denomination]

(i) No. of face cards = 12

$P(\text{face card}) = \frac{12}{40} = \frac{3}{10}$ **Ans.**

(ii) Again, even numbered cards are 2, 4, 8 and 10 each of heart (red) and diamond (red).

$\therefore$ Total even numbered red cards = 4 × 2 = 8

$P(\text{even numbered red card}) = \frac{8}{40} = \frac{1}{5}$

Ans.

(b) $x - \frac{18}{x} = 6$

$\Rightarrow \frac{x^2 - 18}{x} = 6$

$\Rightarrow x^2 - 18 = 6x$

or $x^2 - 6x - 18 = 0$

Since middle term cannot be splitted. So compare with $ax^2 + bx + c = 0$

$a = 1, b = -6, c = -18$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-18)}}{2\times 1}$$

$$= \frac{6 \pm \sqrt{36 + 72}}{2} = \frac{6 \pm \sqrt{108}}{2}$$

$$= \frac{6 \pm \sqrt{6\times 6\times 3}}{2}$$

$$= \frac{6 \pm 6\sqrt{3}}{2}$$

$= 3 \pm 3\sqrt{3} = 3 \pm 3(1{\cdot}732)$

$= 3 \pm 5{\cdot}196$

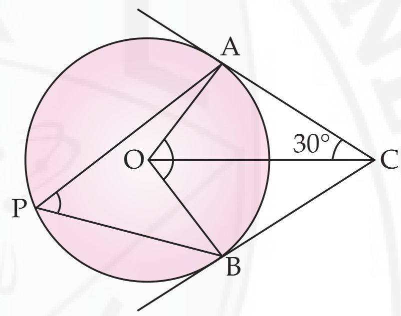
$x = 3 + 5{\cdot}196$ or $x = 3 - 5{\cdot}196$

$x = 8{\cdot}196$ $x = -2{\cdot}196$

$x = 8{\cdot}2$ (2 sig. fig.) or $x = -2{\cdot}2$ (2 sig. fig.)

Ans.

(c) Given, a circle with centre OCA and CB are tangent to it. ∠OAC = ∠OBC = 90° and ∠ACO = 30°.



(i) Since, Δ ACO ≅ Δ OBC = 30° (By SSS)

(AC = BC, AO = OB and OC is common)

$\therefore$ ∠ACO = ∠BCO = 30° **Ans.**

(ii) ∠OAC = ∠OBC = 90°

∠ACO = 30° (given)

∠AOC = ∠BOC ($\because$ ΔACO ≅ ΔBOC)

$\therefore$ ∠AOC ∠BOC = 180° – (90° + 30°)

($\because$ sum of the 3 angles of a Δ is 180°)

∠AOC = 180° – 120°

∠AOC = 60°

$\therefore$ ∠AOB = ∠AOC + ∠BOC

= 60° + 60°

∠AOB = 120° **Ans.**

(iii) $\angle APB = \frac{1}{2}\angle AOB$

$= \frac{120°}{2} = 60°$ **Ans.**

($\because$ Angle subtended at the remaining part of the circle is half the angle subtended at the centre.)

Solution 3.

(a) **(i)** P = ₹ 2500, n = 2 years, *i.e,* 24 months

Total deposited amount = ₹ 2500 × 24

= ₹ 60,000

Maturity amount = ₹ 66,250

$\therefore$ The interest paid by the bank

$$= ₹\,(66{,}250 - 60{,}000)$$
$$= ₹\,6{,}250 \qquad \textbf{Ans.}$$

(ii)
$$I = \frac{P \times n(n+1)}{2 \times 12} \times \frac{r}{100}$$
$$6250 = \frac{2500 \times 24 \times 25}{2 \times 12} \times \frac{r}{100}$$
$$r = \frac{6250}{25 \times 25} = 10\% \text{ p.a.}$$

$\therefore$ Rate of interest is 10% p.a. **Ans.**

(c) Let A $= (1, 3) = (x_1, y_1)$, B $= (4, b) = (x_2, y_2)$, C $= (a, 1) = (x_3, y_3)$ and G $= (4, 3) = (x, y)$

Coordinates of centroid

$$x = \frac{x_1 + x_2 + x_3}{3} \quad \text{and} \quad y = \frac{y_1 + y_2 + y_3}{3}$$
$$4 = \frac{1 + 4 + a}{3} \quad \text{and} \quad 3 = \frac{3 + b + 1}{3}$$
$$12 - 5 = a \quad \text{and} \quad 9 - 4 = b$$
$$a = 7 \quad \text{and} \quad b = 5 \qquad \textbf{Ans.}$$

Solution 4.

(a) $2x - 5 \le 5x + 4 < 11, x \in I$

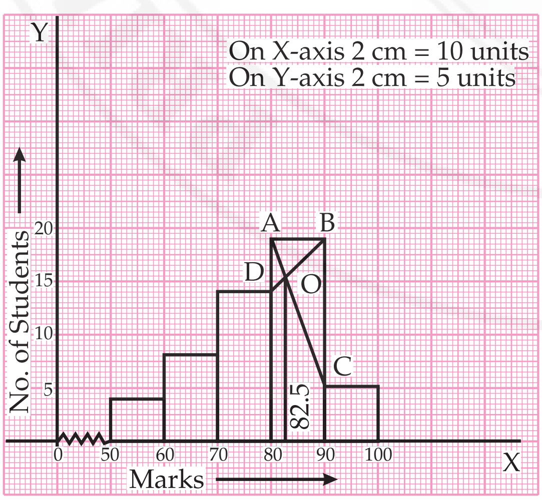
$2x - 5 \le 5x + 4$	$5x + 4 < 11$
$2x - 5x \le 4 + 5$	$5x < 11 - 4$
$-3x \le 9$	$5x < 7$
$3x \ge -9$	$x < \frac{7}{5}$
$x \ge -3$	
$-3 \le x$ ...(i)	$x < 1\frac{2}{5}$ ...(ii)

From (i) and (ii), $-3 \le x < 1\frac{2}{5}$, $x \in I$

$\therefore$ Solution set $= \{-3, -2, -1, 0, 1\}$ **Ans.**

−4 −3 −2 −1 0 1 2

(c)



From the graph, the bar with the maximum height is of 80–90 interval. Join A to C and B to D. They meet at O. Drop perpendicular from O on X-axis. It meets at 82.5.

$\therefore$ mode = 82.5 **Ans.**

SECTION—B

Solution 5.

(b) No. of spheres formed

$$= \frac{\text{Volume of given cone}}{\text{Volume of sphere of radius 0.5 cm}}$$

$$\left[\begin{array}{l} \text{Volume of cone} = \frac{1}{3}\pi r_1^2 h_1 \\ \text{Volume of sphere} = \frac{4}{3}\pi r_2^3 \end{array}\right]$$

$$= \frac{\frac{1}{3}\pi(5)^2 \times 8}{\frac{4}{3}\pi(0{\cdot}5)^3} = \frac{25 \times 8}{4 \times 0{\cdot}5 \times 0{\cdot}5 \times 0{\cdot}5}$$
$$= \frac{25 \times 2 \times 10 \times 10 \times 10}{5 \times 5 \times 5} = 400 \qquad \textbf{Ans.}$$

(c) Given, ABCD is a parallelogram, with A(x, y), B (5, 8), C (4, 7) and D (2, – 4)

(i) As diagonals of a parallelogram bisect each other.

$\therefore$ E is midpoint of BD as well as AC.

$$\text{Coordinates of E} = \left(\frac{x+4}{2}, \frac{y+7}{2}\right)$$

(Using coordinates of A and C)

$$\text{Coordinates of E} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{5+2}{2}, \frac{8-4}{2}\right)$$

(Using coordinates of B and D)

$$= \left(\frac{7}{2}, 2\right)$$

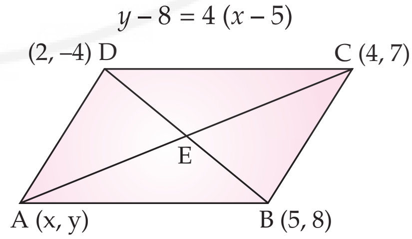
On comparing, $\frac{x+4}{2} = \frac{7}{2}$ and $\frac{y+7}{2} = 2$

$\Rightarrow$ $x = 3$ and $y = -3$

$\therefore$ Coordinates of A are (3, – 3). **Ans.**

(ii) Equation of diagonal BD,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
$$\Rightarrow \quad y - 8 = \frac{-4-8}{2-5}(x - 5)$$
$$\Rightarrow \quad y - 8 = \frac{-12}{-3}(x - 5)$$
$$\Rightarrow \quad y - 8 = 4(x - 5)$$



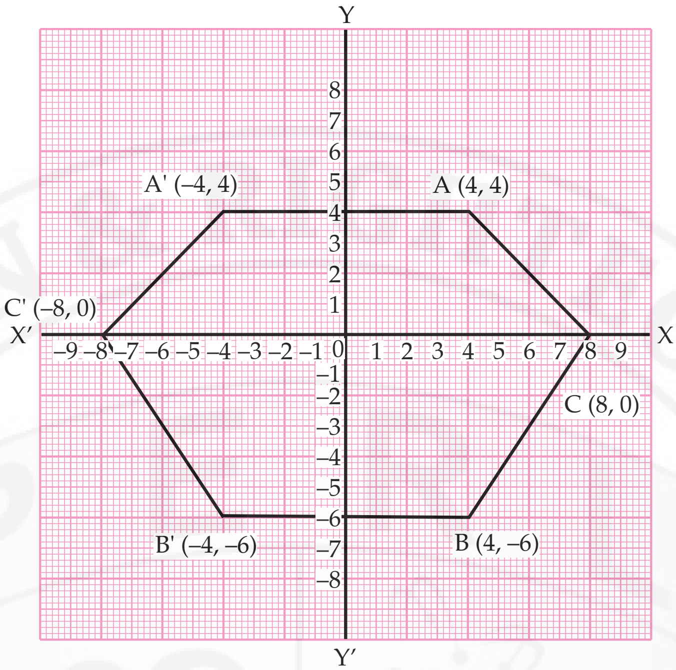
$$\Rightarrow \quad y - 8 = 4x - 20$$
$$\Rightarrow \quad 4x - y - 12 = 0 \qquad \textbf{Ans.}$$

Solution 6.

(a) **(i)** and, **(ii)** see the given graph. **Ans.**

(iii) A′ (– 4, 4), B′ (– 4, – 6), C′ (– 8, 0) **Ans.**

(iv) AA′ C′ B′ BC is a hexagon. **Ans.**



Solution 7.

(a) Given, $\dfrac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}} = \dfrac{9}{1}$

Using componendo and dividendo

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}+\sqrt{3x+4}-\sqrt{3x-5}}{\sqrt{3x+4}+\sqrt{3x-5}-\sqrt{3x+4}+\sqrt{3x-5}} = \frac{9+1}{9-1} = \frac{10}{8} = \frac{5}{4}$$

$$\frac{2\sqrt{3x+4}}{2\sqrt{3x-5}} = \frac{5}{4}$$

$$\Rightarrow \quad \frac{3x+4}{3x-5} = \frac{25}{16}$$

(Squaring both sides)

$$\Rightarrow \quad 48x + 64 = 75x - 125$$

$$\Rightarrow \quad 75x - 48x = 125 + 64$$

$$27x = 189$$

$$\Rightarrow \quad x = \frac{189}{27} = 7$$ **Ans.**

(b) Given, $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$

and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$A^t = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

$$A^t.B = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$A^t.B = \begin{bmatrix} 8-1 & -4+3 \\ 20-3 & -10+9 \end{bmatrix}$$

$$A^t.B = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix} \quad \ldots(i)$$

$$BI = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BI = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} \quad \ldots(ii)$$

From equation (i) and (ii)

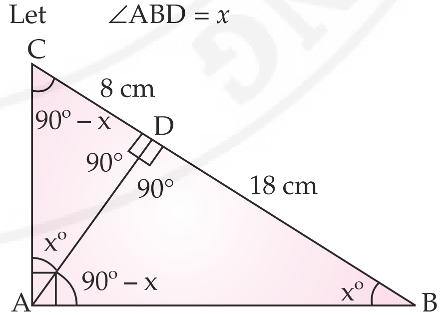
$$A^t.B + BI = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$A^t.B + BI = \begin{bmatrix} 7+4 & -1-2 \\ 17-1 & -1+3 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & -3 \\ 16 & 2 \end{bmatrix}$$ **Ans.**

(c) **(i)** Given, Δ ABC right angled at A, AD ⊥ BC, BD = 18 cm and CD = 8 cm.

Let $\angle ABD = x$



So, $\angle ACD = \angle ACB = 90 - x$ …(i)

($\because \angle BAC = 90°$)

Also $\angle BAD = 90 - x$ …(ii)

($\because \angle ADB = 90°$)

Now, in ΔADB and ΔCDA

$\angle ADB = \angle CDA$

$= 90°$ each (Given)

From (i) and (ii),

$\angle BAD = \angle ACD = 90 - x$

$\therefore$ $\Delta ADB \sim \Delta CDA$ (By AA axiom)

Hence Proved.

(ii) $\because$ $\Delta ADB \sim \Delta CDA$ [Proved in (i)]

$\therefore$ $\frac{AD}{BD} = \frac{CD}{AD}$

(Corresponding sides of similar triangles are proportional)

or $AD^2 = BD \times CD \Rightarrow AD^2 = 18 \times 8$

(BD = 18, CD = 8, given)

$\Rightarrow$ $AD^2 = 144$

$AD = 12$ cm **Ans.**

(iii) $\frac{\text{Area of } \Delta ADB}{\text{Area of } \Delta CDA} = \frac{BD^2}{AD^2}$

[Area theorem of similar triangles]

$= \frac{18^2}{12^2} = \frac{18 \times 18}{12 \times 12}$

$= \frac{3 \times 3}{2 \times 2} = \frac{9}{4}$

$\Rightarrow$ Area (ΔADB) : Area (ΔCDA) = 9 : 4. **Ans.**

Solution 8.

(a) (i)

C.I.	f	x	$u = \frac{x-A}{h}$ where $h = 5$	fu
50–55	5	52·5	– 3	– 15
55–60	20	57·5	– 2	– 40
60–65	10	62·5	– 1	– 10
65–70	10	67·5 = A	0	0
70–75	9	72·5	1	9
75–80	6	77·5	2	12
80–85	12	82·5	3	36
85–90	8	87·5	4	32
	$\Sigma f = 80$			$\Sigma fu = 24$

Mean $(\overline{X}) = A + \frac{\Sigma fu}{\Sigma f} \times h$

(h = length of C.I. = 5)

$= 67·5 + \frac{24}{80} \times 5 = 67·5 + 1·5$

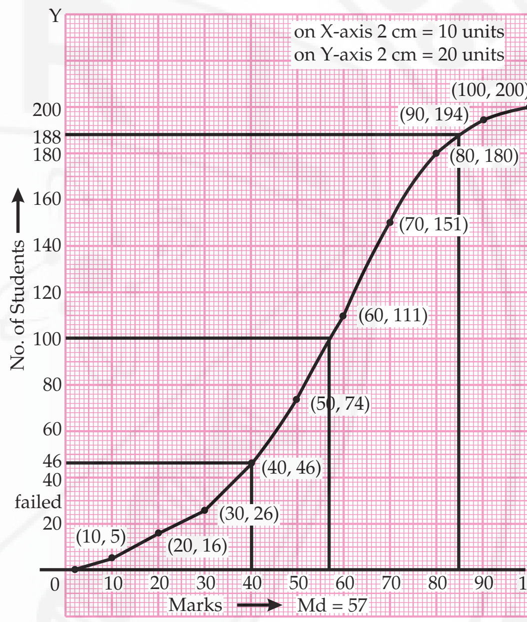
$= 69$ **Ans.**

(ii) Modal class = 55–60

(Class with highest frequency) **Ans.**

(b)

Marks	f	$c.f.$
0–10	5	5
10–20	11	16
20–30	10	26
30–40	20	46
40–50	28	74
50–60	37	111
60–70	40	151
70–80	29	180
80–90	14	194
90–100	6	200



(i) From the graph :

Median $= \left(\frac{n}{2}\right)^{th}$ observation

$= \left(\frac{200}{2}\right)^{th}$ observation

$= 100^{th}$ observation

$= 57$ **Ans.**

On graph, draw a perpendicular from X-axis at 40 marks to the ogive. The point from where line touches ogive drop a perpendicular on the Y-axis. The point where it touches Y-axis is the answer.

(ii) No. of students who failed = 46 **Ans.**

(iii) Number of students who secured grade one = 200 – 188 = 12 **Ans.**

Solution 9.

(a) (i) Given, investment = ₹ 52,000

Nominal value of one share = ₹ 100

Market value of one share

= ₹ 100 – 20 = ₹ 80

Number of shares $= \frac{\text{Investment}}{\text{Market value}}$

$= \frac{52,000}{80} = 650$

Dividend on one share

= Rate of Dividend

× Nominal value of one share

$= \frac{8}{100} \times 100 = ₹\ 8$

∴ Annual dividend = No. of shares

× Dividend on one share

$= 650 \times ₹\ 8 = ₹\ 5,200$ **Ans.**

(ii) Market value of 1 share

= ₹ 100 + ₹ 20 = ₹ 120

Selling price of 650 shares

= ₹ 120 × 650 = ₹ 78,000

Profit earned including his dividend

= Selling value

+ Dividend – Investment

= ₹ 78,000 + ₹ 5,200

– ₹ 52,000

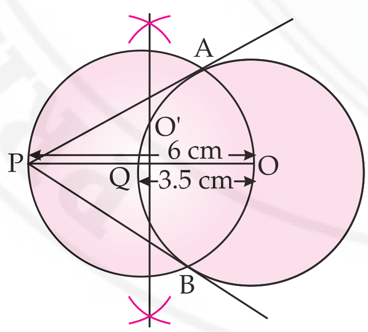
= ₹ 31,200 **Ans.**

(b) Steps of construction :

1. Taking O as centre and OQ as radius equals 3·5 cm, draw a circle.
2. Mark a point P from O at a distance of 6 cm.
3. Draw perpendicular bisector of OP.
4. Draw a circle with O' as a centre which cuts the another circle at points A and B.
5. Join PA and PB.

PA and PB are the required tangents

PA = PB = 4·8 cm



(c) To prove,

(cosec A – sin A) (sec A – cos A) · $\sec^2$ A = tan A

L.H.S. = (cosec A – sin A) (sec A – cos A) · $\sec^2$ A

$= \left(\frac{1}{\sin A} - \sin A\right) \cdot \left(\frac{1}{\cos A} - \cos A\right) \cdot \frac{1}{\cos^2 A}$

$\left[\sec A = \frac{1}{\cos A}, \operatorname{cosec} A = \frac{1}{\sin A}\right]$

$= \left(\frac{1-\sin^2 A}{\sin A}\right) \times \left(\frac{1-\cos^2 A}{\cos A}\right) \times \frac{1}{\cos^2 A}$

$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} \times \frac{1}{\cos^2 A}$

$\begin{bmatrix} 1-\sin^2 A = \cos^2 A \\ 1-\cos^2 A = \sin^2 A \end{bmatrix}$

$= \frac{\sin A}{\cos A} = \tan A$

= R.H.S. **Hence Proved.**

Solution 10.

(a) Given, 6 is mean proportional between x and y.

⇒ x, 6, y are in continued proportion

$\Rightarrow \quad \frac{x}{6} = \frac{6}{y}$

$\Rightarrow \quad xy = 36$

$\Rightarrow \quad x = \frac{36}{y}$ ...(i)

Also, 48 is third proportional of x and y (Given)

⇒ x, y, 48 are in continued proportion.

$\Rightarrow \quad \frac{x}{y} = \frac{y}{48}$

$\Rightarrow \quad y^2 = 48x$ ...(ii)

From (i) $\quad y^2 = 48 \times \frac{36}{y}$

$\Rightarrow \quad y^3 = 48 \times 36$

Taking cube root on both sides

$\Rightarrow \quad \sqrt[3]{y^3} = \sqrt[3]{12 \times 12 \times 12}$

$\Rightarrow \quad y = 12$

And $\quad x = \frac{36}{y}$

$= \frac{36}{12}$

$= 3$

∴ The numbers are 3 and 12. **Ans.**

(c) Let, the height of the building be h

In Δ BCD,

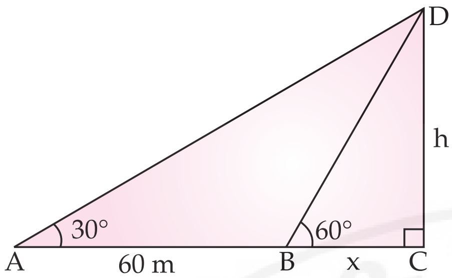
$\frac{h}{x} = \tan 60°$

$\Rightarrow \quad \frac{h}{x} = \sqrt{3}$

$\Rightarrow \quad h = \sqrt{3}x$ ...(i)

In Δ ACD,

$\frac{h}{x+60} = \tan 30°$



$\Rightarrow \quad \dfrac{h}{x+60} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad h\sqrt{3} = x + 60$

$\Rightarrow \quad \sqrt{3}\,x\ \sqrt{3} = x + 60$ [Using (i)]

$\Rightarrow \quad 3x - x = 60$

$\Rightarrow \quad 2x = 60$

$\Rightarrow \quad x = 30$ m

Now, from (i)

$h = \sqrt{3}\ x$

$h = 30 \times \sqrt{3}$

$= 30 \times 1{\cdot}732$

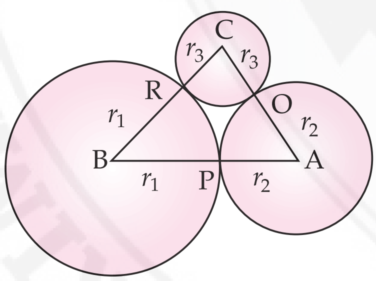
Height = 51·96 m = 52 m (rounded off)

The height of the building is 52 m. **Ans.**

Solution 11.

(a) Given, AB = 10 cm, BC, = 8 cm, AC = 6 cm

Let the radii of three circles be r_1, r_2 and r_3 (shown in fig.)



Now, AB = $r_1 + r_2 = 10$ ...(i)

AC = $r_2 + r_3 = 6$ ...(ii)

BC = $r_3 + r_1 = 8$ ...(iii)

Adding equations (i), (ii) and (iii)

$2(r_1 + r_2 + r_3) = 10 + 6 + 8 = 24$

$r_1 + r_2 + r_3 = 12$ ...(iv)

Subtract (i) from (iv)

$\Rightarrow \quad r_2 = 12 - 10 = 2$ cm

Subtract (ii) from (iv)

$\Rightarrow \quad r_1 = 12 - 6 = 6$ cm

Subtract (iii) from (iv)

$\Rightarrow \quad r_2 = 12 - 8 = 4$ cm

Therefore, the radius of 3 circles are 2 cm, 6 cm and 4 cm. **Ans.**

(b) Let, number of children be x

Share of each child = ₹ $\dfrac{480}{x}$

Now, number of children = $x + 20$

$\because$ share of each child = ₹ $\dfrac{480}{x+20}$

Now, According to the question

$\dfrac{480}{x} - \dfrac{480}{x+20} = 12$

$\Rightarrow \quad \dfrac{480x + 9600 - 480x}{x(x+20)} = 12$

$\Rightarrow \quad 9600 = 12x(x + 20)$

$\Rightarrow \quad 800 = x^2 + 20x$

$\Rightarrow \quad x^2 + 20x - 800 = 0$

$\Rightarrow \quad x^2 + 40x - 20x - 800 = 0$

$\Rightarrow \quad x(x + 40) - 20(x + 40) = 0$

$\Rightarrow \quad (x - 20)(x + 40) = 0$

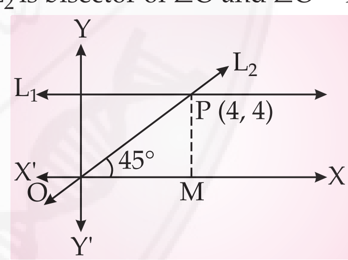
Either $x = 20$

or $x = -40$ (not possible)

$\therefore$ Number of children is 20. **Ans.**

(c) Equation of L_1 is $y = 4$ (given)

(i) As L_2 is bisector of $\angle O$ and $\angle O = 90°$



$\Rightarrow$ L_2 is inclined at an angle of 45° with XX′

$\therefore$ Slope of $L_2 = m = \tan 45° = 1$ **Ans.**

(ii) Slope of $L_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4-0}{x-0}$

Where, $x_1 = 0,\ y_1 = 0$

$x_2 = x,\ y_2 = y$ (eq. of L_1 is $y = 4$)

Slope of $L_2 = 1$ (using (i))

$\Rightarrow \quad 1 = \dfrac{4}{x}$

$\Rightarrow \quad x = 4$

So, coordinates of P are (4, 4) **Ans.**

(iii) Equation of L_2

$y - 4 = 1(x - 4)$ $\left(\begin{array}{l}L_2 \text{ pass through } (4,4) \\ \text{and has slope } m = 1\end{array}\right)$

$y - 4 = x - 4$

or $x = y$

or $x - y = 0$ **Ans.**